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955 L'ENFANT PLAZA NORTH, S.W., WASHINGTON, D.C. 20024

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- A Determination of Optimum SPS
Propellant Offloading Including
Consideration of Spacecraft Com-
pletion of Translunar Injection

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DATE- June 22, 1970

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AUTHOR(S)- R. J. Stern

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ABSTRACT

Two methods for increasing the probability of successful translunar injection (TLI) and the probability of mission success are analyzed. The first method involves offloading SPS propellant to reduce launch vehicle injected payload and satisfy launch vehicle injection requirements. An optimal value for propellant offloaded is determined which maximizes the composite probability of a successful translunar injection by the SIVB and the probability of the successful completion of all required spacecraft maneuvers in the presence of trajectory dispersions and performance uncertainties. Using current 2σ launch vehicle baseline commitments applied to J missions with a control weight spacecraft, it is shown that approximately $3/4$ of the maximum SPS propellant offload above the spacecraft 3σ requirements should be offloaded to achieve maximum assurance of mission success. For a heavier spacecraft this fraction increases and in some cases maximum probability of success is achieved by maximum propellant offloading.

The second method consists of launching the spacecraft with full SPS tanks and, for a certain range of off-nominal launch situations where complete injection by the launch vehicle (SIVB) is not possible, utilizing available spacecraft propellant margins to complete the translunar injection. It is shown that this method increases the probability of successful injection and mission success. The magnitude of the increase is strongly dependent upon how soon after SIVB engine cutoff the SPS corrective maneuver can be performed. For a sufficiently early corrective maneuver this method could be used to satisfy the two-sigma injection requirement for certain missions where that constraint is violated for fully-loaded SPS tanks. For later times the method allows a reduction in the minimum required propellant offloaded to meet the injection requirement. For any amount of propellant offloading, the second method always provides a higher probability of mission success.

It is assumed that normal probabilities can be associated with the SIVB flight performance reserve propellant and the spacecraft dispersion propellant budget.

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INTRODUCTION

Many of the currently proposed lunar missions have sizeable Service Propulsion System (SPS) propellant margins with full tanks over and above the propellant needed for required mission maneuvers, a contingency ΔV budget of 600 ft/sec, and three-sigma (3σ) trajectory dispersions and performance uncertainties. On the other hand, J-mission CSM and LM weights with full spacecraft propellant tanks may exceed the launch vehicle injected payload capability. Two methods are proposed to satisfy mission energy requirements and increase the probability of successful injection.

One of these methods is the reduction of SPS propellant margin prior to launch. The question then arises as to how much propellant should be offloaded. It is possible to offload the minimum amount of SPS propellant required to satisfy the launch vehicle injected weight limit. However, since the current SIVB flight performance reserves correspond to a two-sigma (2σ) probability level of propellant non-depletion and since the spacecraft has a propellant margin above the nominal dispersion budget which provides for a 3σ probability level, it may be desirable to offload more than the minimum propellant required. This would have the effect of decreasing the injected weight, thereby increasing the probability of a successful translunar injection (TLI) by the launch vehicle at the expense of carrying less SPS propellant margin above 3σ requirements, which decreases the probability of successfully performing the required spacecraft maneuvers in the presence of dispersions and performance uncertainties. In this memorandum an optimal value for offloaded SPS propellant is determined that maximizes a composite probability which is an indicator of overall mission success. In the event of improved launch vehicle performance prediction, the composite probability is maximized using lower values of SPS propellant offloaded. The numerical results generated in this memorandum serve primarily to illustrate the optimization method. More specific results can be obtained once operational trajectory data become available for a specific mission.

The method of offloading described above assumes that injection is completed by the SIVB stage of the launch vehicle. In a second proposed method the mission is flown with full SPS tanks. For a certain range of off-nominal injection situations exceeding

the propellant capability of the SIVB, injection is performed by burning the SIVB engine to propellant depletion and completing the injection maneuver with spacecraft propellant reserves as soon as possible after SIVB engine cutoff. This method is also investigated and the probabilities of successful injection and mission success associated with it are determined. It is also shown that both methods, i.e. offloading and spacecraft completion of injection can be combined to satisfy injection requirements and increase the probability of mission success.

ANALYSIS

• Composite Probability of Mission Success

The nominal Apollo mission can be thought of as a sequence of maneuvers all of which must be performed successfully. Therefore, we could attach a compound probability of overall success to the mission given by

$$P_{\text{mission success}} = P_{L/V} \cdot P_{\text{CSM}} \cdot P_{\text{LM DESCENT}} \cdot P_{\text{LM ASCENT}} \cdot \dots \quad (1)$$

where $P_{L/V}$ denotes the probability of a successful injection by the launch vehicle, P_{CSM} denotes the probability of successful Command and Service Module Performance* (midcourse corrections, LOI, TEI, etc.), $P_{\text{LM DESCENT}}$ denotes the probability of a successful LM descent, etc. With regard to offloading SPS propellant or completing injection with the spacecraft, only $P_{L/V}$ and P_{CSM} are affected; therefore, for present purposes all other factors may be considered constant. Consequently we may maximize a measure of the probability of a successful mission by maximizing the composite probability P_C where

$$P_C = P_{L/V} \cdot P_{\text{CSM}} \quad (2)$$

The probabilities $P_{L/V}$ and P_{CSM} are functions of launch vehicle and spacecraft dispersions and the available propellant allocated for such dispersions. If it is assumed that normal probabilities

* P_{CSM} is a conditional probability, i.e., it is conditional upon a successful injection by the launch vehicle.

may be attached to the launch vehicle and spacecraft dispersion allowances without introducing significant error, then P_C can be computed given the σ levels corresponding to the launch vehicle and spacecraft dispersion budgets. The determination of these σ levels is considered in the following two sections.

• Launch Vehicle Sigma Level

Launch vehicle performance is concerned with vehicle flight from lift-off to final SIVB engine cutoff. The SIVB stage rocket engine nominally performs the final boost into earth parking orbit and the translunar injection maneuver. A portion of the SIVB propellant budget is designated as the Flight Performance Reserve (FPR). This propellant reserve is distinct from the Flight Geometry Reserve (FGR) and provides for a successful injection in the presence of errors due to uncertainty in mass, propulsion, aerodynamics, and environment. The determination of a typical FPR is described in References 1 and 2. Current launch vehicle baseline performance includes a 2σ SIVB flight performance budget, which corresponds to a one-sided* normal probability = .97725 of propellant non-depletion during injection. Launch vehicle performance provides 2σ capability for a baseline payload to be injected from a 90 NM earth parking orbit to a reference energy of $-8.05 \times 10^6 \text{ ft}^2/\text{sec}^2$ plus a FGR allowance of 32.8 ft/sec. Baseline payloads for J missions are presented in Reference 3.

For a specific mission several factors and options may be used to increase the injected payload above the baseline value while still maintaining the 2σ flight performance reserve. These include increased performance due to temperature and wind variations, propellant savings from a mission specific energy less than the reference value and the use of a portion of the flight geometry reserve.** Therefore, for a given mission and launch vehicle, there exists a specific payload that may be injected with a 2σ probability of success. This weight denoted as $W_{2\sigma}$ is presented in Table I for several sample missions.

*Only those dispersions causing SIVB propellant consumption above the nominal value may result in unsuccessful injections. The same one-sided probabilities apply to the spacecraft.

**Twenty-three of the 32.8 ft/sec FGR was budgeted to account for variations in earth-moon geometry. On a mission specific basis this allotment is not required (see Reference 4 for a discussion of the FGR).

The maximum σ probability level for a complete injection by the SIVB with an injected weight W_I different from $W_{2\sigma}$ is approximated by

$$\sigma_{L/V} = 2.0 + \left[\frac{\partial \text{ lb SIVB propellant}}{\partial \text{ lb SIVB injected inert weight}} \times \frac{\partial \text{ sigma level}}{\partial \text{ lb SIVB propellant}} \right] \cdot (W_{2\sigma} - W_I) \quad (3)$$

The exchange ratio: lb SIVB propellant/lb inert weight at injection is approximately .90. The value of SIVB propellant per sigma is estimated at 966 lbs, which would correspond to a 3σ FPR of 2898 lbs. Using these values,

$$\sigma_{L/V} = 2.0 + .000942 (W_{2\sigma} - W_I) \quad (4)$$

The actual injected weight W_I is obtained in terms of SPS propellant offloaded by means of the relation,

$$W_I = W_{\text{FULL SPS}} - \text{PROPELLANT OFFLOADED} \quad (5)$$

where $W_{\text{FULL SPS}}$ = the injected weight corresponding to full SPS tanks. These relations are employed to determine $\sigma_{L/V}$ as a function of SPS propellant offloaded.

• Spacecraft Sigma Level

The SPS propellant budget contains a nominal reserve designated for dispersions. This budget provides the spacecraft with 3σ capability to perform all required maneuvers in the

presence of initial state vector errors, navigation uncertainties, guidance errors, and performance uncertainties (References 5 and 6). A typical SPS 3σ dispersion budget for a J mission is approximately 615 lbs or 205 lbs/ σ .

Many proposed J missions have sizeable SPS margins above 3σ requirements. We can consider any existing propellant margin as being added to the nominal dispersion budget thereby providing a probability of successful spacecraft performance in excess of the 3σ level. The available σ level is approximated by

$$\sigma_{\text{CSM}} = 3.0 + (\text{propellant margin above } 3\sigma) / 205 \text{ lbs}/\sigma \quad (6)$$

The end-of-mission propellant margin above 3σ requirements, F_{eom} is related to propellant offloaded, F_{O} , by means of (see Appendix A.1)

$$F_{\text{eom}} = F_{\text{eomf}} \cdot \left(1 - \frac{F_{\text{O}}}{F_{\text{om}}} \right) \quad (7)$$

where F_{om} = maximum possible propellant offloaded

F_{eomf} = end-of-mission propellant margin with full SPS tanks

Values of the end-of-mission SPS propellant margin with full tanks and maximum offloading limits are presented in Table I for several sample missions. It can be noted that for all missions the maximum propellant that can be offloaded is approximately twice the end of mission propellant margin for a fully-tanked mission. This is a result of diminished propellant requirements as the spacecraft weight decreases due to offloading. Therefore,

$$F_{\text{eom}} \approx F_{\text{eomf}} - F_{\text{O}}/2 \quad (8)$$

RESULTS FOR SPS PROPELLANT OFFLOADING (Injection by the SIVB Only)

Using the preceeding relations the composite probability P_C was computed as a function of propellant offloaded for six sample missions to three lunar landing sites. Two weight models were considered for each mission: the first corresponding to current control weights and the second including the effects of possible growth and increased payload. These weight models are described in Table II. Spacecraft weights for actual missions will probably be between the control and growth weights. The mission independent ΔV budget employed in the determination of the propellant reserve with full SPS tanks and the maximum permissible offloading is described in Table III.

The results are presented in Figures 1-4 where composite probability is presented as a function of the fraction of the maximum possible propellant offloaded for missions to Marius Hills, Descartes, Copernicus and Hadley. It should be noted that actual propellant offloaded corresponding to a specified fractional value depends upon the maximum value that can be offloaded (above 3σ requirements) and is therefore mission dependent. It may be seen that for the control weight spacecraft the composite probability is maximized by offloading approximately $3/4$ of the maximum possible in all cases. This optimal fraction increases for the growth weight spacecraft, and for Copernicus maximizing the composite probability requires that the maximum possible propellant be offloaded. This increase in propellant offloaded to obtain a maximum P_C is due to the increased spacecraft weight.

As the injected weight increases, the constraining factor on the composite probability is that due to the launch vehicle. This factor is maximized by offloading as much as possible thereby reducing the injected weight. The increase in composite probability from the fully-tanked spacecraft to the case of optimal offloading is significant. For example a control-weight mission to Hadley (7/23/74) has a composite probability = .9934 with full SPS tanks and a composite probability = .9999982 for the optimally offloaded case.

The 2σ launch vehicle constraint can be illustrated by a horizontal line in Figures 1-4. When the fraction of SPS propellant offloaded is small, the factor P_{CSM} is near unity and the composite probability is essentially determined by the factor $P_{L/V}$. Therefore, the intersection of this line with the composite probability curves defines the minimum fraction of SPS propellant that must be offloaded to maintain the 2σ launch vehicle requirement. If the 2σ line does not intersect the probability curve for a given mission, then that mission can be flown with full SPS tanks and not violate the 2σ launch vehicle constraint. However, it is clear from the cases illustrated that, even where possible, flying the mission with full tanks does not maximize the composite probability of mission success.

When the 2σ launch vehicle injected weight capability is sufficiently large and/or there exists a sizeable SPS propellant margin for the mission, the factor $P_{L/V}$ will approach unity when the maximum possible propellant is offloaded. When this occurs the composite probability is essentially determined by the factor P_{CSM} . It may be seen that, for all of the control weight missions and most of the growth weight missions, the composite probability approaches the 3σ level when the maximum possible propellant is offloaded. This is due to the fact that in the maximum offloaded state the SPS margin is zero and the spacecraft is operating at the 3σ or nominal dispersion budget level.

As might be expected, the maximum value of composite probability for a given mission with a growth weight spacecraft is less than that for a control weight spacecraft. This decrease can be considered as a penalty paid in the composite probability, which is directly related to the probability of mission success, as a result of the additional spacecraft weight.

In Figures 5 and 6 the σ probability levels for the spacecraft and launch vehicle are presented as functions of the fraction of SPS propellant offloaded for four sample missions with a control and growth weight spacecraft respectively. The optimal values of propellant offloaded are depicted by vertical lines. It may be noted that maximum composite probability occurs when the σ levels for the launch vehicle and spacecraft are approximately equal. An exception to this occurs when the $\sigma_{L/V}$ and σ_{CSM} curves for a given mission do not intersect e.g. Copernicus (1/28/74, growth weight) in which case the optimal offloading ratio equals one.

INCOMPLETE INJECTION BY THE SIVB STAGE

• Analysis

From the previous results it is clear that offloading SPS propellant can result in an increased probability of mission success. For the missions considered where the fully-tanked spacecraft violates the 2σ launch vehicle constraint, offloading can be employed to satisfy that constraint. The intuitive disadvantage of offloading lies in the fact that, if launch vehicle performance is near nominal, we have given up a propellant reserve which might be useful in a later contingency situation.

An alternative method of increasing the probability of injection and mission success is to fly the mission with full SPS tanks and to allow for the possibility of completing TLI

with the SPS for a certain set of off-nominal situations. For a given mission, launch vehicle, and a specified amount of propellant offloaded, it has been established that there exists a maximum SIVB σ probability level (equation 4) for which complete injection by the SIVB engine is possible, utilizing the entire flight performance reserve. Suppose that the launch vehicle performance is sufficiently off-nominal such that a completed translunar injection would require more than the provided FPR. In other words, an SIVB propellant deficit, F_{SIVBd} , exists and it is not possible for the SIVB to complete the TLI maneuver. At this point a decision to abort the mission or perform an alternate mission might be required (Reference 7). However, it may be possible to perform the TLI maneuver by burning the SIVB engine to propellant depletion resulting in an incomplete injection and completing the injection maneuver with available spacecraft propellant as soon as possible after SIVB engine cutoff.

An estimation of the spacecraft propellant required to complete the translunar injection in the event of incomplete injection by the SIVB is described in Appendix B. It is shown for example that approximately four lbs of spacecraft propellant are required for each lb of SIVB propellant deficit when the corrective spacecraft maneuver is performed four hours after SIVB engine shutdown. Required spacecraft propellant for other corrective maneuver times is presented in Table IV. It is important to note, however, that the spacecraft propellant available for the corrective maneuver is greater than the end of mission spacecraft propellant reserve above 3σ requirements. In fact the maximum available spacecraft propellant, $F_{SPSI(MAX)}$, is shown (Appendix A.2) to be equal to

$$F_{SPSI(MAX)} = \left(\frac{F_{om}}{F_{eomf}} \right) \cdot F_{eom} \approx 2 F_{eom} \quad (9)$$

where F_{om} , F_{eomf} are defined on Page 5 and F_{eom} is given by equation (7). For current missions and spacecraft weights the propellant available at injection is approximately twice the end of mission reserve. This effect is due to the fact that by utilizing the spacecraft propellant we are essentially performing an offloading operation which reduces spacecraft weight and propellant requirements for the later phases of the mission.

• Increased Composite Probability

When the option of completing TLI with spacecraft propellant is included the composite probability of mission success defined by equation (2) increases. This is true since for all cases where the SIVB sigma propellant level* is less than the limiting value defined in equation (4), injection by the SIVB alone is possible and in addition there exists another set of further off-nominal cases where injection can be completed by the spacecraft and subsequent phases of the mission performed successfully.

The increase in composite probability is readily described as a sum of individual terms. Each term consists of the probability that the equivalent** SIVB sigma propellant level lies in a certain range multiplied by the probability of successful spacecraft performance in that event. The probability of successful spacecraft performance in each term is therefore conditional upon the equivalent SIVB sigma propellant level, σ_{SIVB} . When the sigma propellant level required for injection is less than the limiting value for complete injection by the SIVB, the probability of successful spacecraft performance is found from equation (6) and is independent of the sigma propellant level required. When the required sigma propellant level exceeds $\sigma_{\text{L/V}}$ the probability of successful spacecraft performance is diminished in accordance with the amount of SPS propellant required to complete the injection. The increased composite probability, P_C , is therefore expressible as

$$P_C = P_{\text{L/V}} \cdot P_{\text{CSM}} + \sum_{\sigma_{\text{SIVB}} = \sigma_0}^{\sigma_{\text{SIVB}} = \sigma_I} P(\sigma_{i-1} < \sigma_{\text{SIVB}} < \sigma_i) \cdot P_{\text{CSM}_i} \quad (10)$$

*Sigma propellant level is the required propellant relative to the nominal propellant expenditure in terms of its sigma value. For example an SIVB propellant requirement of 2898 lbs. above the nominal requirement is the same as a 3 σ propellant level (966 lbs/ σ).

**The use of the word equivalent denotes the fact that SIVB propellant above the sigma value defined by equation (4) is not available. However, the use of spacecraft propellant is equivalent to assuming that an additional SIVB reserve exists.

where $\sigma_0 = \sigma_{L/V}$, $\sigma_i = \sigma_{i-1} + \Delta\sigma^*$ and $\sigma_I =$ the maximum sigma level for a successful injection utilizing all available SPS propellant above the 3σ spacecraft reserves (Appendix B). The terms P_{CSM_i} are calculated by determining the amount of SPS propellant expended to complete injection as a function of σ_{SIVB} as described in Appendix B, redefining the SPS propellant margin and using equation (6).

• Results

Using the above analysis the composite probability for injection by the SIVB alone or by the SIVB and spacecraft was determined. The results are illustrated in Figures 7-12 for several sample missions with control- and growth-weight spacecrafts. Composite probabilities corresponding to possible injection by the SIVB alone or by the SIVB and spacecraft are presented as a function of fraction of SPS propellant offloaded for several times at which the SPS injection completion maneuver might be performed. The time designated as zero hours represents a hypothetical limiting case where injection is completed by the spacecraft immediately after SIVB engine cutoff. For comparative purposes the composite probability corresponding to injection by the SIVB only is again presented. The composite probabilities for fully-loaded SPS tanks lie on the vertical line corresponding to zero fraction of SPS propellant offloaded.

It is clear from Figures 7-12 that the option of completing injection with the spacecraft can increase the composite probability significantly depending upon how soon after SIVB cutoff the injection can be completed. The significant decrease in composite probability over a time span of several hours after SIVB cutoff is due to the rapid decrease in spacecraft velocity which in turn results in greater required propellant expenditures to make up the energy deficit caused by premature SIVB cutoff. It can also be seen that the optimal value of SPS propellant offloaded varies from zero when injection is performed immediately after SIVB cutoff to the optimal value for injection by the SIVB only, as the time after SIVB cutoff increases and the spacecraft reserve exerts a negligible effect on the injection. It may be noted, however, that for times of injection completion greater than two hours the optimal offload fraction is essentially the same as in the case where injection is performed by the SIVB only.

* $\Delta\sigma$ is a suitable step size used in the numerical calculation of the summation term in equation (10).

Allowing injection completion by the spacecraft increases the sigma probability level for a successful TLI. This increased sigma level, σ_I , is presented in Figures 13-15 as a function of fraction of SPS propellant offloaded for different spacecraft corrective maneuver times. The cases illustrated are growth-weight missions where the two-sigma injection* constraint is violated with full SPS tanks. The sigma level for successful injection using the SIVB only, $\sigma_{L/V}$ is also presented for comparative purposes. The figures indicate that the sigma level for successful injection can be increased significantly depending upon how soon after SIVB cutoff injection is completed by the spacecraft. For example, the figures indicate that the injection sigma level for growth-weight fully-tanked missions to Descartes and Hadley can be increased by approximately 0.7 by performing the corrective maneuver two hours after SIVB cutoff. This is equivalent to an additional SIVB performance reserve of $.7\sigma \times 966 \text{ lbs}/\sigma = 676 \text{ lbs}$. For lighter spacecraft, greater increases are possible arising from larger SPS propellant margins. Therefore, if the injection completion is performed soon enough after SIVB cutoff the two-sigma injection constraint can be satisfied for certain fully-tanked SPS missions. For later times of injection completion the method can be used to reduce the minimum amount of offloaded propellant required to satisfy the injection constraint. It should also be noted that the results of Figures 13-15 depend only on the launch vehicle sigma level and the available spacecraft propellant to complete the injection and are not dependent upon the association of normal probabilities with the spacecraft dispersion budget.

DISCUSSION

• Examination of Methods

From the previous results it is possible to draw certain conclusions about the relative merits of offloading SPS propellant prior to launch and allowing for completing of injection by the spacecraft in certain off-nominal situations.

From Figures 1-6 it is clear that offloading the optimal fraction of SPS propellant satisfies the 2σ injection requirement and significantly increases the probability of mission success. The disadvantage of SPS propellant offloading arises from the fact that if launch vehicle performance is near-nominal, a propellant reserve has been lost which might be required in a contingency situation.

*The current two-sigma launch vehicle injection requirement is replaced here by a general two-sigma injection requirement since injection is no longer performed solely by the launch vehicle.

Allowing for completion of injection by the spacecraft for a certain set of off-nominal launch situations can eliminate or at least reduce the minimum offloading requirement to satisfy the 2σ injection constraint, thereby providing additional spacecraft propellant for contingencies in the event of a near nominal launch. It is shown in Figures 13-15 that this option can be used to meet 2σ injection requirements for a growth-weight spacecraft with full SPS tanks if the SPS correction can be made sufficiently soon after SIVB cutoff. If this is not possible then the method at least reduces the fraction of required propellant offloaded to satisfy the injection constraint. From Figures 7-12 it is readily seen that this option increases the probability of mission success for any propellant offloading. However, when this option is applied to a fully-tanked spacecraft, the composite probability is less than that attainable by optimal offloading unless the injection can be completed very soon (< 2 HR) after SIVB cutoff.

It is important to note that both methods can be used in combination. That is, partial offloading and possible spacecraft completion of injection could be employed to satisfy the injection constraint and increase probability of mission success.

- Effect of Improved Launch Vehicle Performance

The previous results were generated using a 2σ baseline payload probability level. In the event of possible improvements in launch vehicle performance, the launch vehicle sigma levels for the baseline injected weights considered here would increase. The effect of an increase in baseline launch vehicle sigma level is illustrated in Figures 16-19 where composite probability is presented as a function of fractional SPS propellant offloaded above 3σ requirements for several values of baseline launch vehicle sigma level. The cases illustrated are control and growth weight spacecraft missions to Descartes and Copernicus. It may be seen from the figures that the effect of a higher baseline launch vehicle sigma level is to increase the composite probability and decrease the optimal offloading fraction.

- Assumption of Normality

The analysis presented herein represents a general method and criteria that can be applied to the determination of optimal SPS propellant offloading and the effectiveness of completing injection with the spacecraft. The specific results in Figures 1-4, 7-12 and 16-19 are keyed to the assumption that that normal probabilities may be attached to the launch vehicle and spacecraft dispersions budgets.

This assumption is probably quite good for the launch vehicle FPR since the assumptions of statistical independence of perturbing variables and linearity between flight parameter variables and perturbation variables* are employed in the determination of the FPR and are shown to be fairly accurate for most error sources (References 1 and 2). The assumption violated most frequently is that of linearity between the perturbations and resulting dispersions. When this occurs positive and negative dispersions are combined separately. Although the method is not completely accurate in defining flight parameter dispersions, it is justified by its ease of implementation and the fact that the accuracy of the method may well be greater than the accuracy to which the perturbations are known.

The spacecraft dispersion budget represents a statistical sum of propellant dispersions for all SPS spacecraft maneuvers. The major contributions arise from the first translunar midcourse correction, LOI, and TEI. To test the assumption of attaching normal probabilities to the spacecraft dispersion budget, Monte Carlo samples of SPS propellant expended for the above three maneuvers were obtained from MSC for a representative mission.** The frequency distributions for the maneuvers were added statistically assuming statistical independence between samples to obtain a frequency distribution for the spacecraft dispersion budget. The cumulative probability for this distribution is presented in Figure 20 along with the cumulative probability for an equivalent normal distribution having the same mean and variance. It may be seen from Figure 20 that the two probability curves are in good agreement. This suggests that the assumption of normal probabilities for the spacecraft dispersion budget is a reasonable one.

CONCLUSIONS

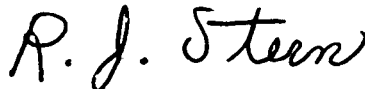
Offloading SPS propellant reserves is a feasible method for satisfying two-sigma injection requirements and increasing the probability of mission success. By assuming normal probabilities for the launch vehicle and spacecraft propellant dispersion budgets it is shown that for a given mission and launch

*When these conditions are met and the perturbing variables are normally distributed, the flight parameter dispersions, e.g. SIVB propellant, are also normally distributed.

**Apollo 13 data.

vehicle there exists an optimal value for SPS propellant offloaded which maximizes probability of mission success. The results for sample missions to Marius Hills, Descartes, Copernicus, and Hadley indicate that for current 2σ launch vehicle baseline commitments and a control-weight spacecraft, approximately $3/4$ of the maximum possible SPS propellant offload above the spacecraft 3σ requirements should be offloaded to guarantee maximum probability of mission success. For a heavier spacecraft this ratio increases and may equal unity in some cases thus requiring maximum propellant offloading.

The option of performing the injection by burning the SIVB to propellant depletion and completing the injection with spacecraft propellant reserves may also be used to increase the sigma level for a successful TLI and the probability of mission success. The extent of this increase is strongly dependent upon how soon after SIVB cutoff the injection can be completed by the spacecraft. For sufficiently early times this method can be used to satisfy injection requirements for certain growth weight missions with full SPS tanks. For later times this option can be used to reduce the minimum required propellant offloaded to satisfy the two-sigma injection constraint.



R. J. Stern

2013-RJS-slr

Attachments

APPENDIX A

A.1 End of Mission Propellant Reserve as a Function of Fuel Offloaded.

Given an end-of-mission propellant margin with initially fully-loaded SPS tanks F_{eomf} , the end-of-mission propellant margin resulting when a given amount of propellant, F_O , is offloaded is given by,

$$F_{eom} = F_{eomf} - F_O + rF_O \quad (A-1)$$

where r is a constant exchange ratio of pounds of SPS propellant used for the mission per pound of spacecraft weight. The last term represents the reduction in propellant required because the spacecraft is lighter by an amount F_O . The exchange ratio r can be defined by noting that $F_{eom} = 0$ when F_O is at its maximum value F_{om} . Therefore

$$r = 1 - \frac{F_{eomf}}{F_{om}} \quad (A-2)$$

and substituting into equation (A-1) we have

$$F_{eom} = F_{eomf} \left(1 - \frac{F_O}{F_{om}} \right) \quad (A-3)$$

as presented in the section entitled Spacecraft Dispersion Budget (equation 7).

A.2 Propellant Available for Completion of the Injection Maneuver by the Spacecraft

The end of mission propellant reserve, F'_{eom} existing after a corrective maneuver by the spacecraft is given by

$$F'_{eom} = F_{eom} - (1-r)F_{SPSI} \quad (A-4)$$

where F_{eom} , r are defined above and F_{SPSI} = the SPS propellant used to complete the translunar injection. The maximum value of available propellant (above 3σ requirements) is obtained by setting $F'_{eom} = 0$.

$$F_{SPSI}(MAX) = F_{eom}/(1-r) \quad (A-5)$$

Substituting for r we obtain,

$$F_{SPSI}(MAX) = \left(\frac{F_{om}}{F_{eomf}} \right) \cdot F_{eom} \quad (A-6)$$

For current mission weights the ratio in parenthesis is approximately two and the available SPS propellant for completing the injection maneuver is therefore approximately twice the end of mission propellant reserve.

APPENDIX B

Estimate of Spacecraft Propellant Required to Complete TLI

Consider an off-nominal launch situation where a complete injection would require more SIVB propellant than is available. This propellant deficit is readily converted to a characteristic velocity deficit by means of the exchange ratio of 10.3 lbs SIVB propellant per ft/sec. The effect of incomplete injection by the SIVB was simulated by subtracting a ΔV deficit from the nominal velocity at TLI while maintaining the nominal injection position, and velocity direction. This impulsive approximation is reasonable since an SIVB propellant reserve of one thousand lbs is expended in approximately two seconds.

The resulting trajectory was then propagated conically for specified time intervals up to ten hours after SIVB cutoff and the position and velocity of the spacecraft was determined. For each position the required velocity to retarget the nominal mission was found based on achieving the nominal time of lunar landing and number of revolutions in lunar orbit, post LOI, and minimizing LOI ΔV . The required velocity and current spacecraft velocity were then differenced to obtain the ΔV to complete injection from which the required SPS propellant was calculated.

The results of this conic simulation are found in Table IV where several exchange ratios relating required SPS propellant to SIVB deficits are presented. The exchange ratio lbs SPS propellant per $\Delta\sigma_{\text{SIVB}}$, where $\Delta\sigma_{\text{SIVB}}$ = the increase in equivalent SIVB σ propellant level that would be required to complete the injection, is of importance since it determines the increase in the sigma level for a successful injection. The maximum increase in $\Delta\sigma_{\text{SIVB}}$ is attained when all the available spacecraft propellant (above 3 σ requirements) $F_{\text{SPSI}(\text{MAX})}$ is employed. Therefore

$$\Delta\sigma_{\text{SIVB}(\text{MAX})} = F_{\text{SPSI}(\text{MAX})} / \text{lbs SPS propellant per } \Delta\sigma_{\text{SIVB}} \quad (\text{B-1})$$

Thus by allowing injection completion by the spacecraft the maximum sigma level for a successful injection is increased by $\Delta\sigma_{SIVB(MAX)}$ and is given by

$$\sigma_I = \sigma_{L/V} + \Delta\sigma_{SIVB(MAX)} \quad (B-2)$$

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2. Saturn V AS-508 H-2 Mission Launch Vehicle Flight System Dispersion Analysis, Document No. D5-15553(I) - 8, Boeing Company, January 22, 1970, Volume I of II.
3. Apollo Mission Performance Presentation, K. E. Martersteck, Bellcomm Memorandum for File B70 03002, Case 310, March 2, 1970.
4. Saturn V. Flight Geometry Reserve Handbook, Boeing Document No. D5-15733 B, October 6, 1967.
5. Apollo 12 Spacecraft Dispersion Analysis, Volume VII, Dispersion Summary, MSC Internal Note 69-FM-292, November 6, 1969.
6. Apollo 11 Spacecraft Dispersion Analysis, Volume III, LOI-1, LOI-2, Plane Change and TEI Dispersions, MSC Internal Note 69-FM-174, June 24, 1969.
7. Final Mission Flight Rules, Apollo 13, Flight Control Division, MSC - 01807, February 12, 1970.

TABLE I

MISSION PROPELLANT RESERVE AND LAUNCH VEHICLE PAYLOAD LIMIT

Landing Site	Launch Date	2 σ Launch Vehicle Injected Payload $W_{2\sigma}$	CONTROL WEIGHT SPACECRAFT*		GROWTH WEIGHT SPACECRAFT**	
			End of Mission SPS Propellant Margin With Full Tanks	Maximum Possible Propellant Offloading	End of Mission SPS Propellant Margin With Full Tanks	Maximum Possible Propellant Offloading
Marius Hills	7/30/71 8/29/71	107,415 107,457	2378 1555	4480 2990	1989 1158	3750 2220
Descartes	1/18/72 2/17/72	106,581 106,381	1546 2145	2964 4045	1149 1756	2204 3310
Copernicus	1/28/74 2/27/74	107,085 106,942	914 1084	1812 2121	510 683	1011 1336
Hadley	7/23/74 8/21/74	107,005 106,972	1658 1492	3169 2870	1262 1094	2414 2106

*Injected weight (full SPS tanks) = 106,500

**Injected weight (full SPS tanks) = 107,481

TABLE II
WEIGHT MODELS

	Control Weight Spacecraft	Growth Weight* Spacecraft
CSM Weight less payload	25,000	25,450
CSM payload	<u>750</u>	<u>1,000</u>
CSM Weight	25,750	26,450
LM Weight less payload	35,150	35,331
LM payload	<u>850</u>	<u>1,000</u>
LM Weight	36,000	36,331
SPS usable propellant	39,410	39,410
SPS unusable propellant	<u>1,190</u>	<u>1,190</u>
Tanked SPS propellant	40,600	40,600
CSM Weight	25,750	26,450
+ LM weight	36,000	36,331
+ SPS propellant	40,600	40,600
+ SLA	<u>4,150</u>	<u>4,100</u>
Injected weight with full SPS tanks	= 106,500	107,481

* From March, 1970 ASSB Meeting

TABLE III

J MISSION INDEPENDENT ΔV'S

EVENT

1	2	3	4	5	6	7	8	9
ΔV (FPS)	HYBRID MANEUVER	TRANSLUNAR MIDCOURSE = 32.9	LOI PLUS *C.C=70 PLUS DOI=75	---	CIRCULARIZATION = 75	CSM PLANE CHANGE	LM RESCUE = 600	TEI + *C.C=70
WEIGHT DROP (LBS)	149	219	64	610 + LM	169	330	299	---

SPS ISP = 313.9

*CONIC CALIBRATION

TABLE IV

SPS PROPELLANT REQUIREMENTS TO COMPLETE TLI

Mission	Time From SIVB Cutoff (hrs)	C O N T R O L W E I G H T				G R O W T H W E I G H T			
		Lbs SPS Propellant Per SIVB ΔV Deficit (lbs/ft per sec)	Lbs SPS Propellant Per Lb SIVB Deficit (lbs/lbs)	Lbs SPS Propellant Per $\Delta\sigma$ SIVB (lbs)	Lbs SPS Propellant Per SIVB ΔV Deficit (lbs/ft per sec)	Lbs SPS Propellant Per Lb SIVB Deficit (lbs/lbs)	Lbs SPS Propellant Per $\Delta\sigma$ SIVB (lbs)		
Marius Hills 7/30/71	0	10.11	0.98	946.7	10.21	0.99	956.3		
	2	33.21	3.22	3114.9	33.54	3.26	3146.2		
	4	41.44	4.02	3886.8	41.86	4.06	3925.9		
	6	47.73	4.63	4476.5	48.21	4.68	4521.5		
	8	53.21	5.17	4990.1	53.75	5.22	5040.4		
	10	58.23	5.65	5461.7	58.84	5.71	5516.7		
Descartes 1/18/72	0	10.11	0.98	946.7	10.21	0.99	956.3		
	2	32.70	3.18	3066.9	33.70	3.27	3161.2		
	4	41.09	3.99	3852.3	41.92	4.07	3932.1		
	6	47.49	4.61	4452.1	48.18	4.68	4519.1		
	8	53.05	5.15	4972.9	53.85	5.23	5050.0		
	10	58.12	5.64	5451.5	58.87	5.72	5530.7		
Copernicus 1/28/74	0	10.11	0.98	946.7	10.21	0.99	956.3		
	2	33.60	3.26	3151.5	33.91	3.29	3183.2		
	4	41.82	4.06	3924.2	42.24	4.10	3963.7		
	6	48.18	4.68	4518.9	48.66	4.72	4564.4		
	8	53.71	5.21	5036.8	54.25	5.27	5087.5		
	10	58.79	5.71	5512.9	59.38	5.76	5568.4		
Hadley 7/23/74	0	10.11	0.98	946.7	10.21	0.99	956.3		
	2	33.70	3.27	3158.8	34.04	3.30	3190.0		
	4	41.83	4.06	3921.9	42.24	4.10	3965.1		
	6	48.03	4.66	4501.6	48.51	4.71	4551.5		
	8	53.43	5.19	5013.5	53.97	5.24	5059.8		
	10	58.34	5.66	5467.6	58.93	5.72	5527.8		

Factors Relating Exchange Ratios

lbs SIVB propellant/ft per second at SIVB cutoff = 10.3

lbs SIVB propellant/ $\Delta\sigma$ SIVB = 966

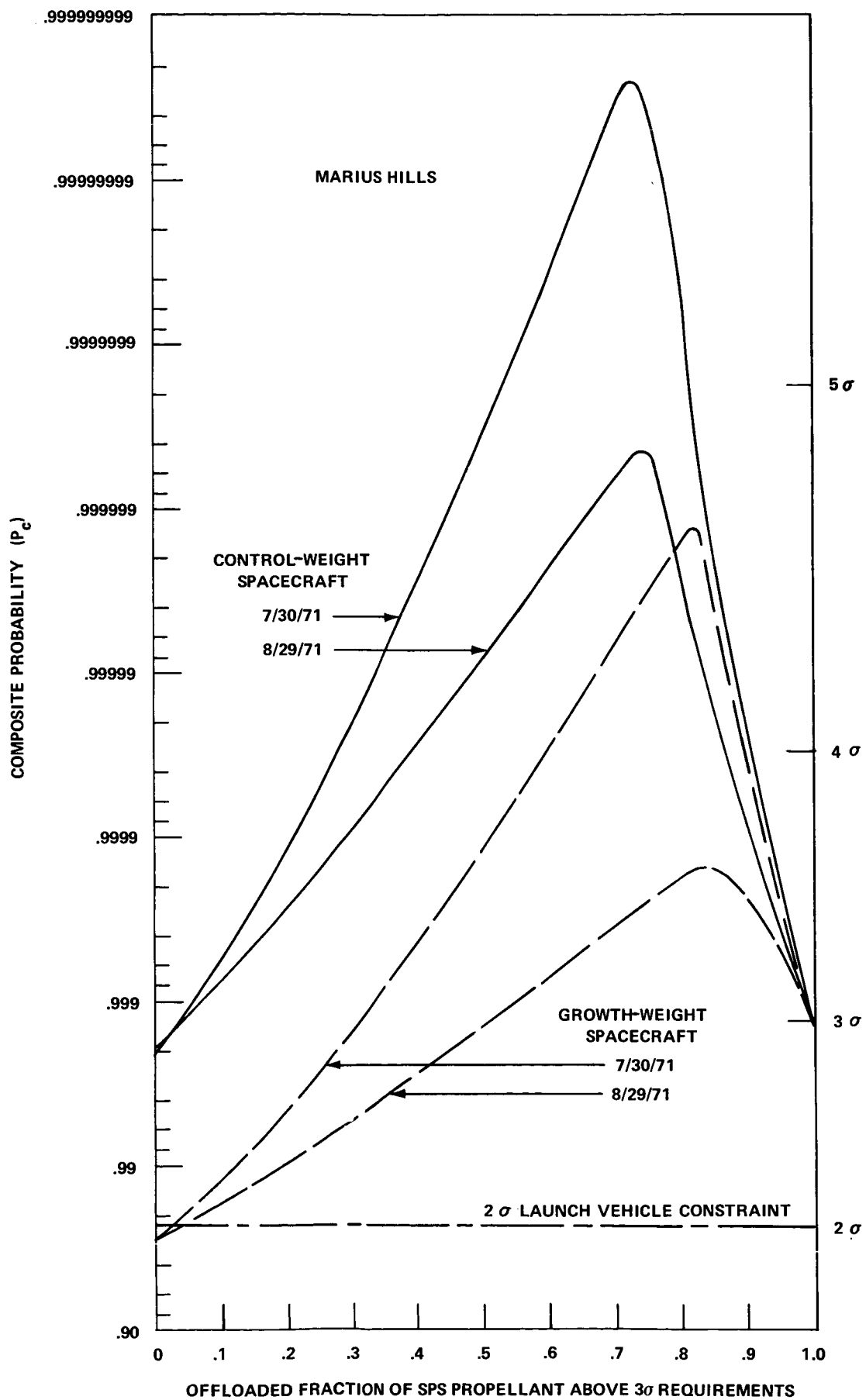


FIGURE 1 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING (INJECTION BY SIVB ONLY)

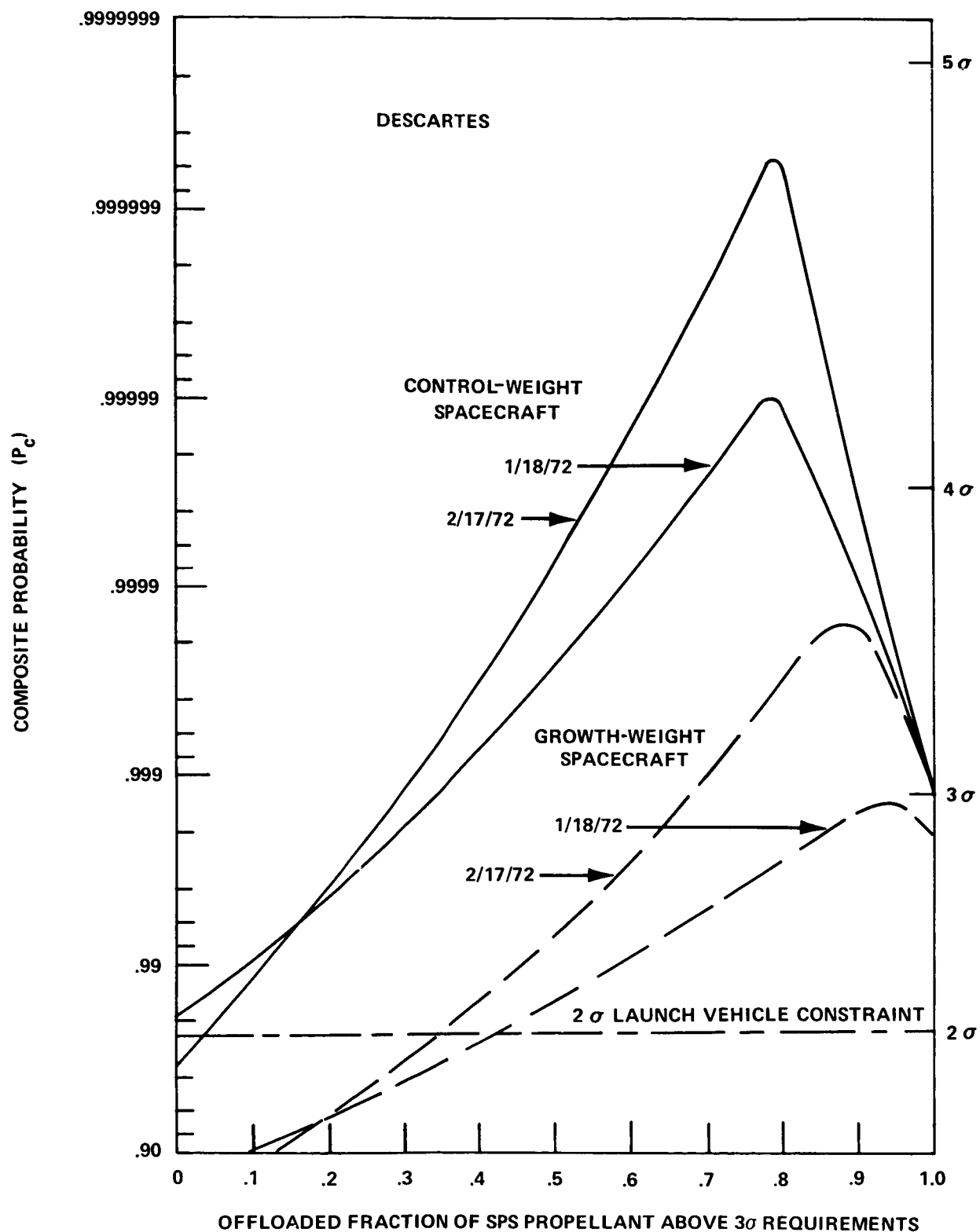


FIGURE 2 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING (INJECTION BY SIVB ONLY)

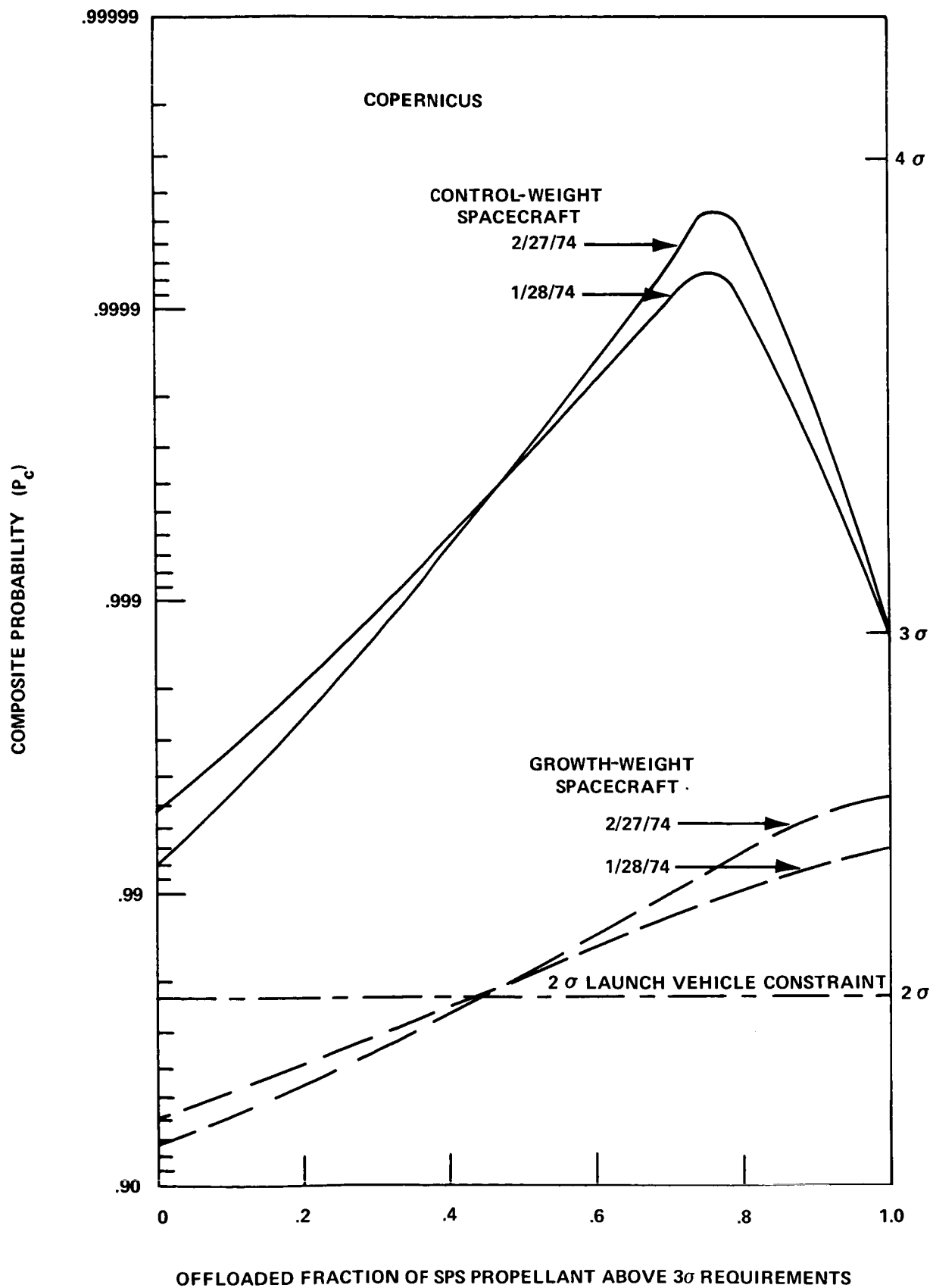


FIGURE 3 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB ONLY)

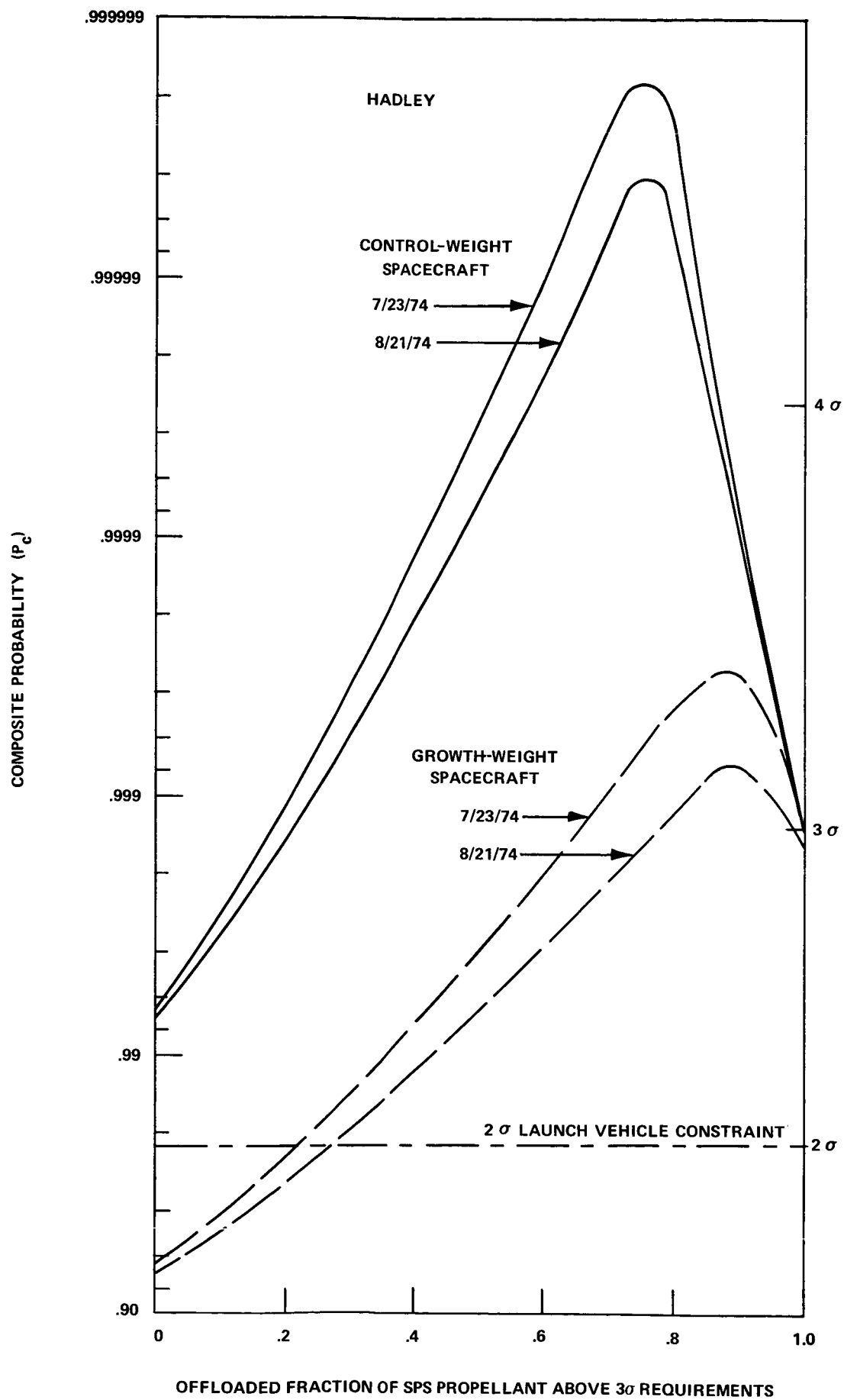


FIGURE 4 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING (INJECTION BY SIVB ONLY)

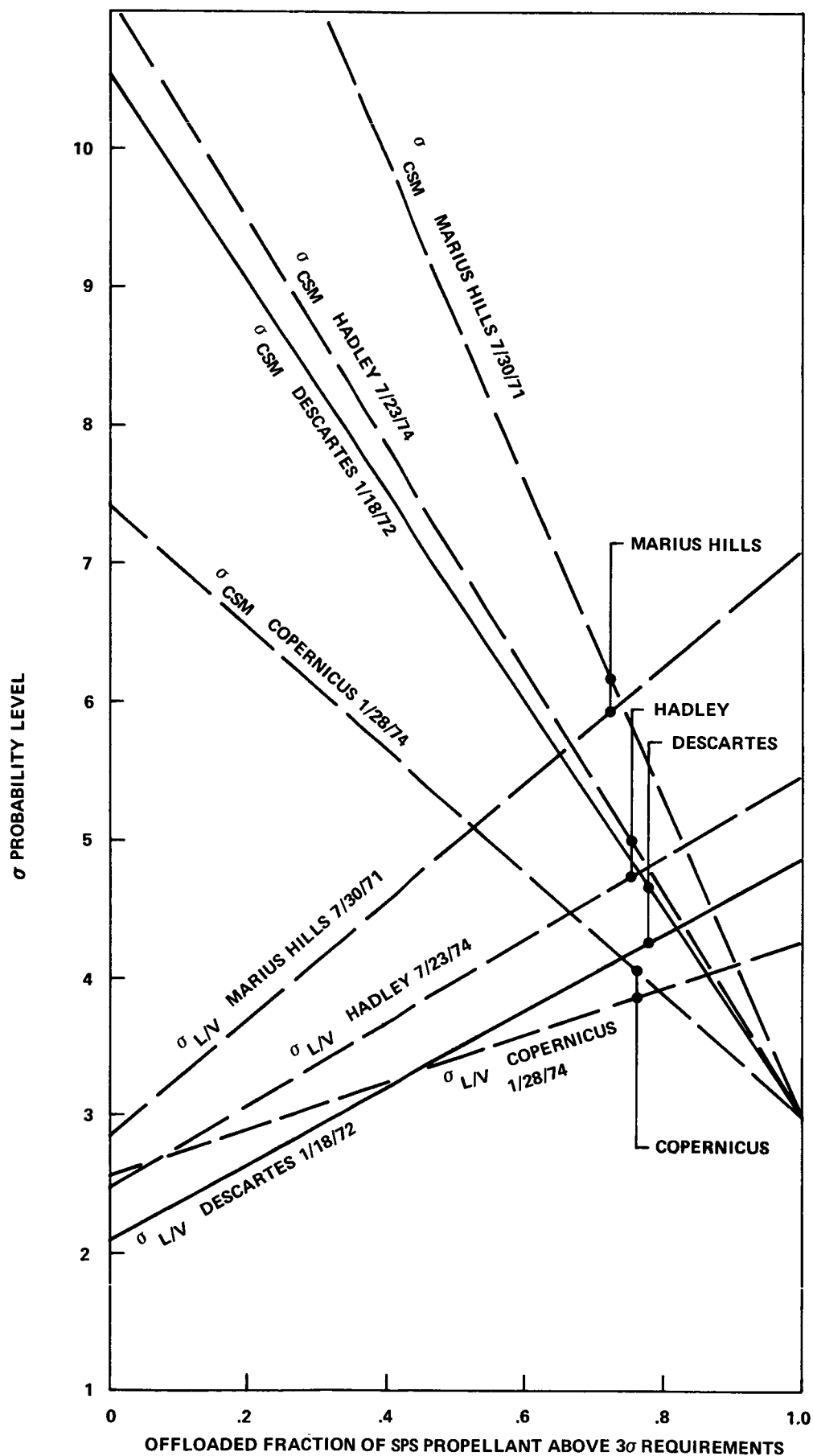


FIGURE 5 - CONTROL-WEIGHT SPACECRAFT AND LAUNCH VEHICLE SIGMA PROBABILITY LEVELS vs. SPS PROPELLANT OFFLOADING.

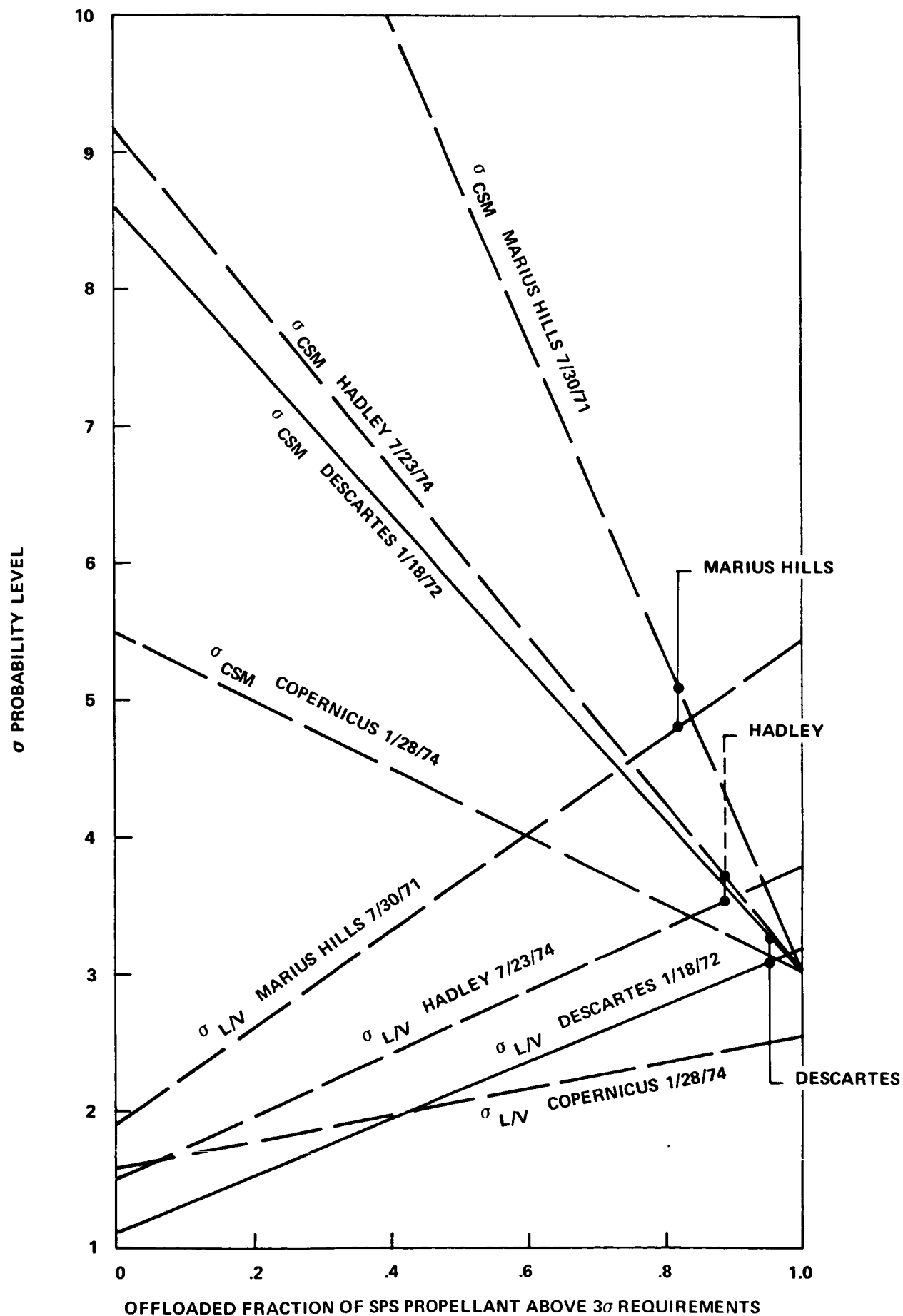


FIGURE 6 - GROWTH-WEIGHT SPACECRAFT AND LAUNCH VEHICLE SIGMA PROBABILITY LEVELS vs. SPS PROPELLANT OFFLOADING.

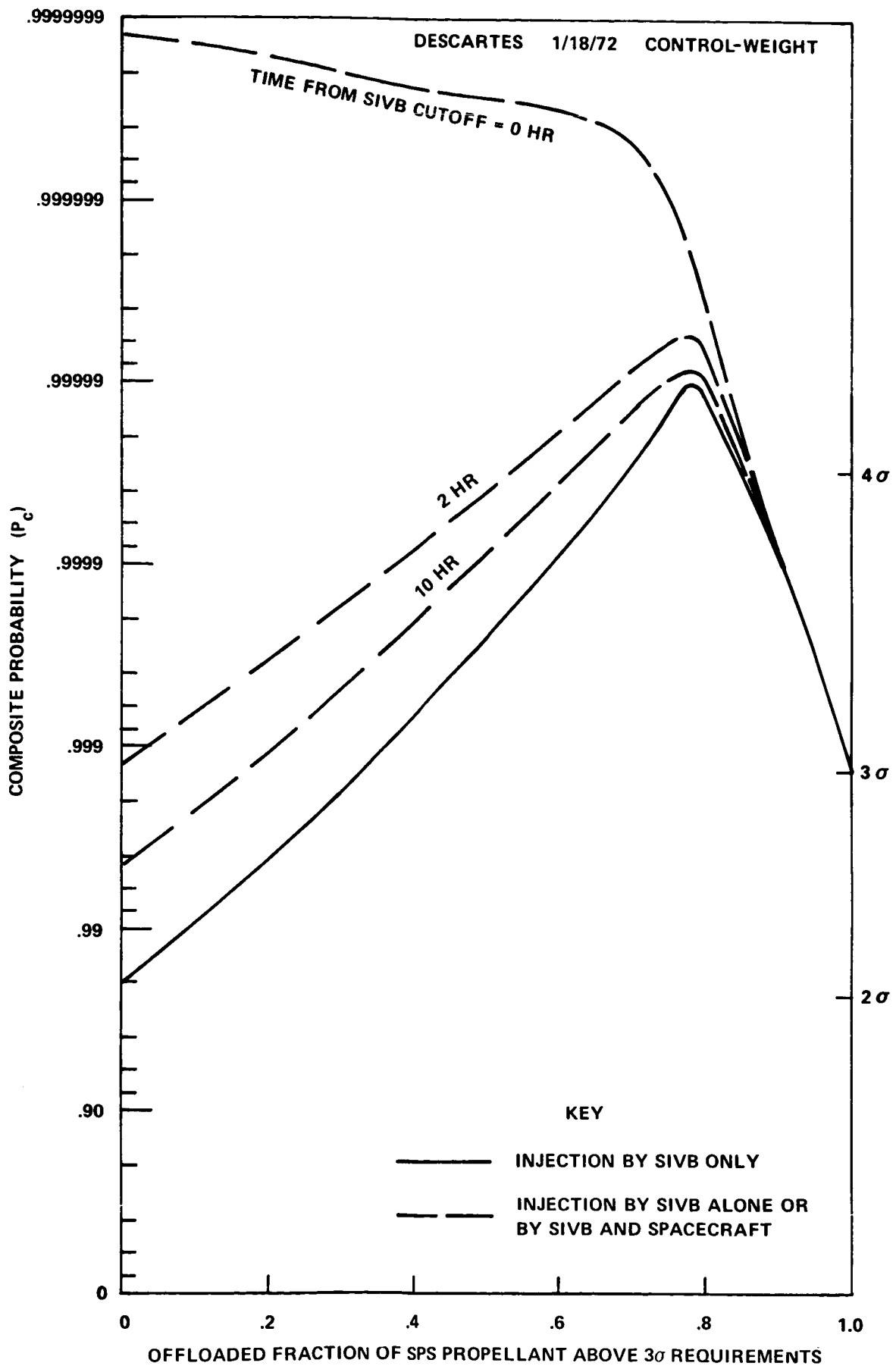


FIGURE 7 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

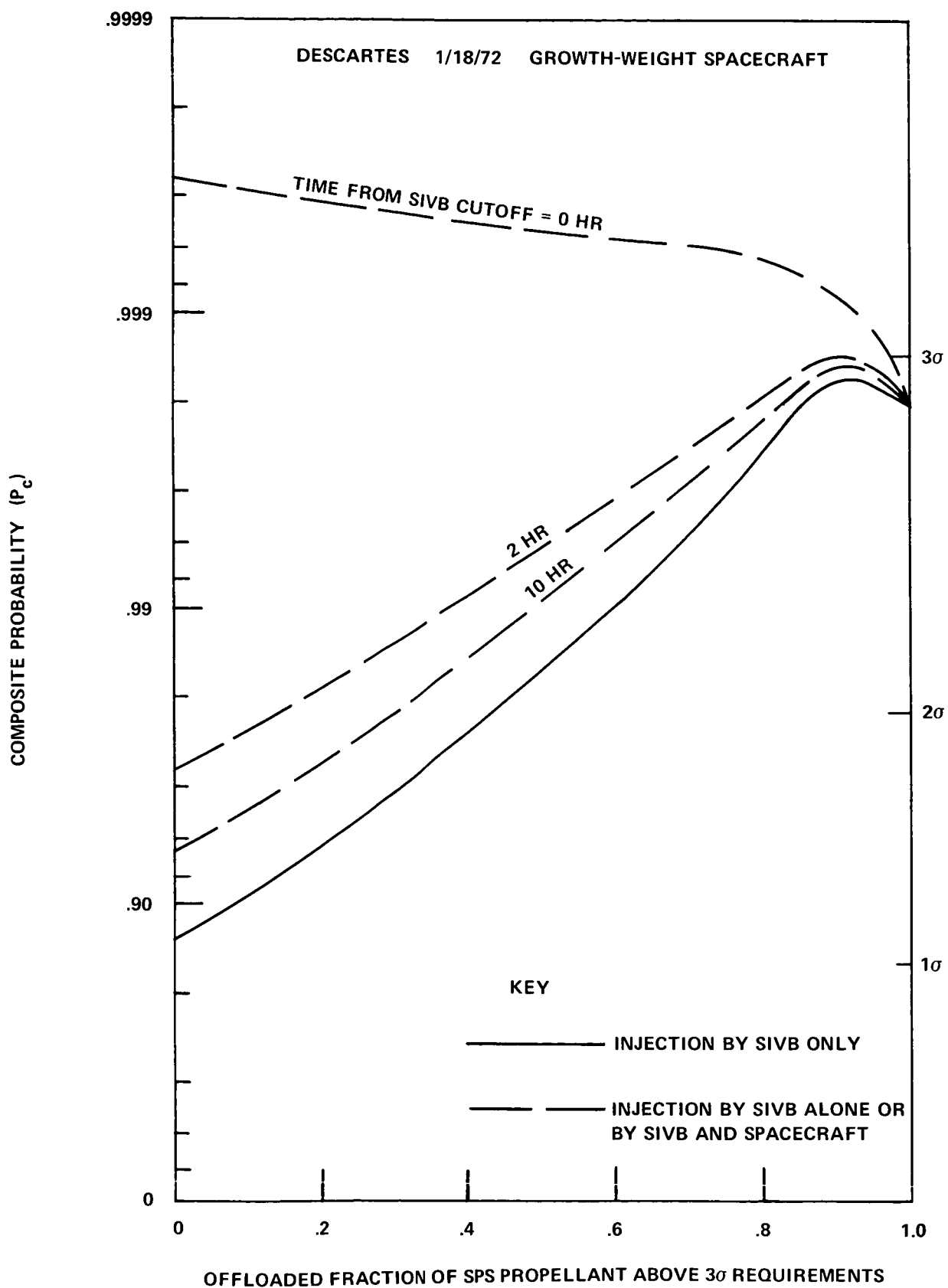


FIGURE 8 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

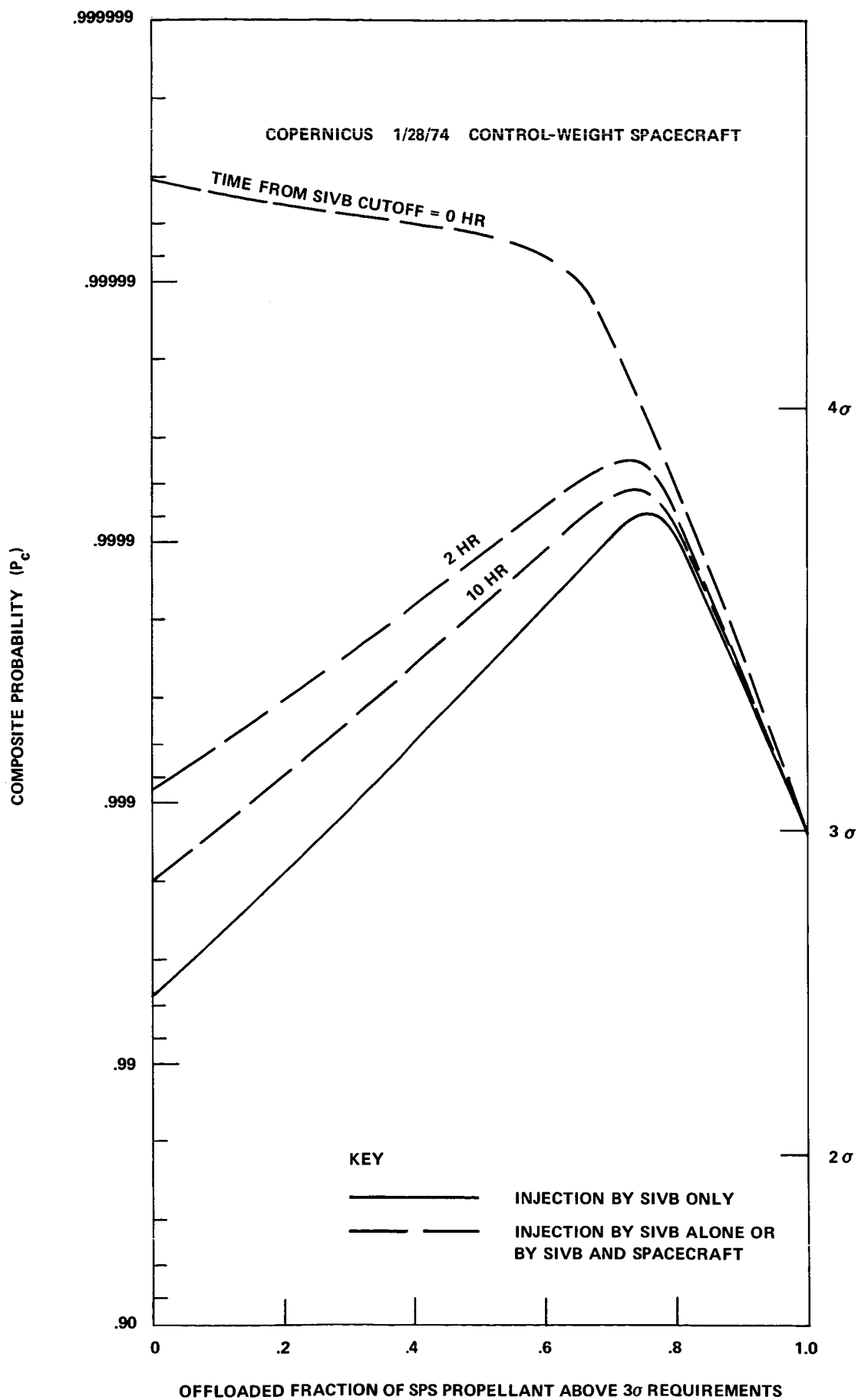


FIGURE 9 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

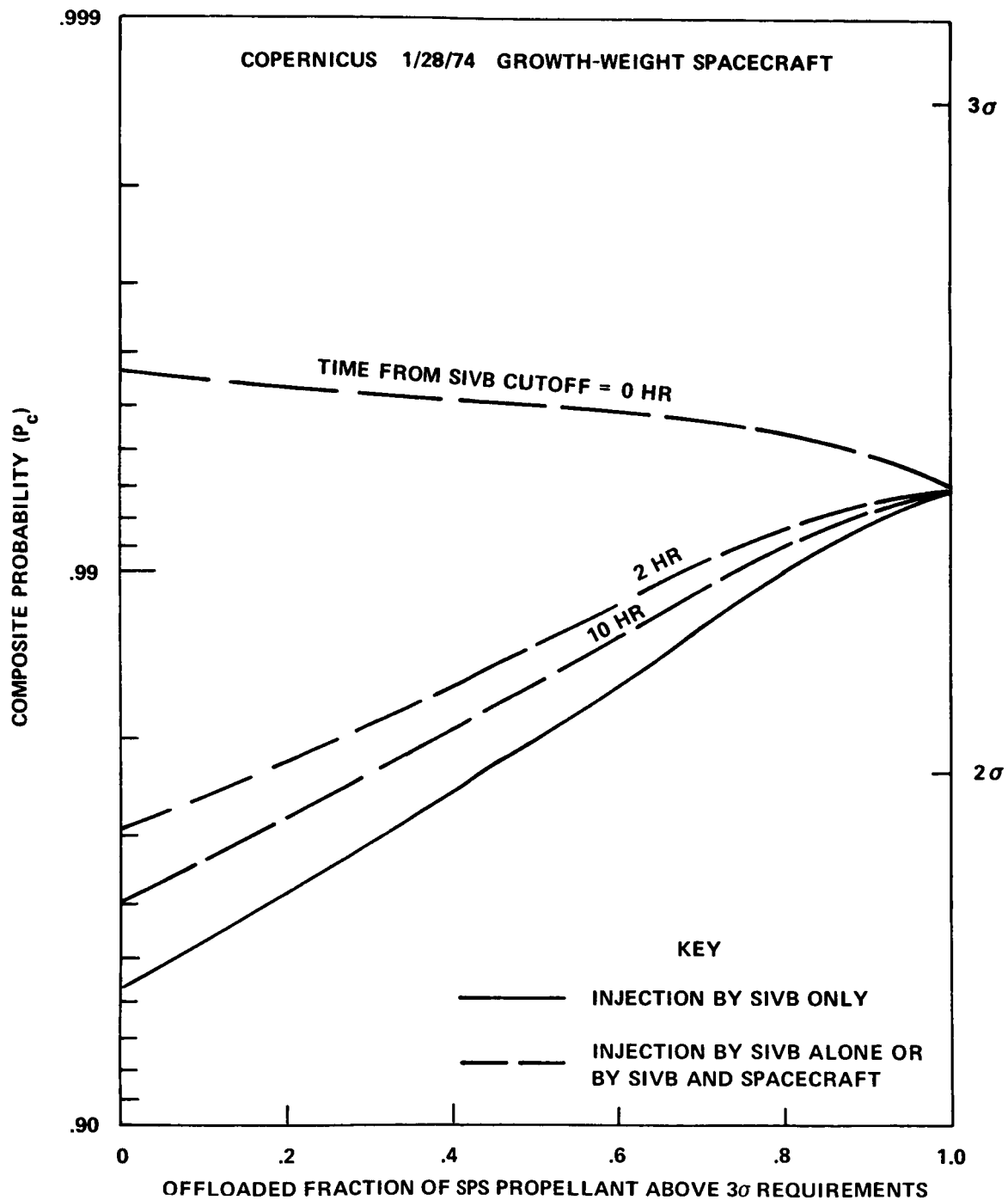


FIGURE 10 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

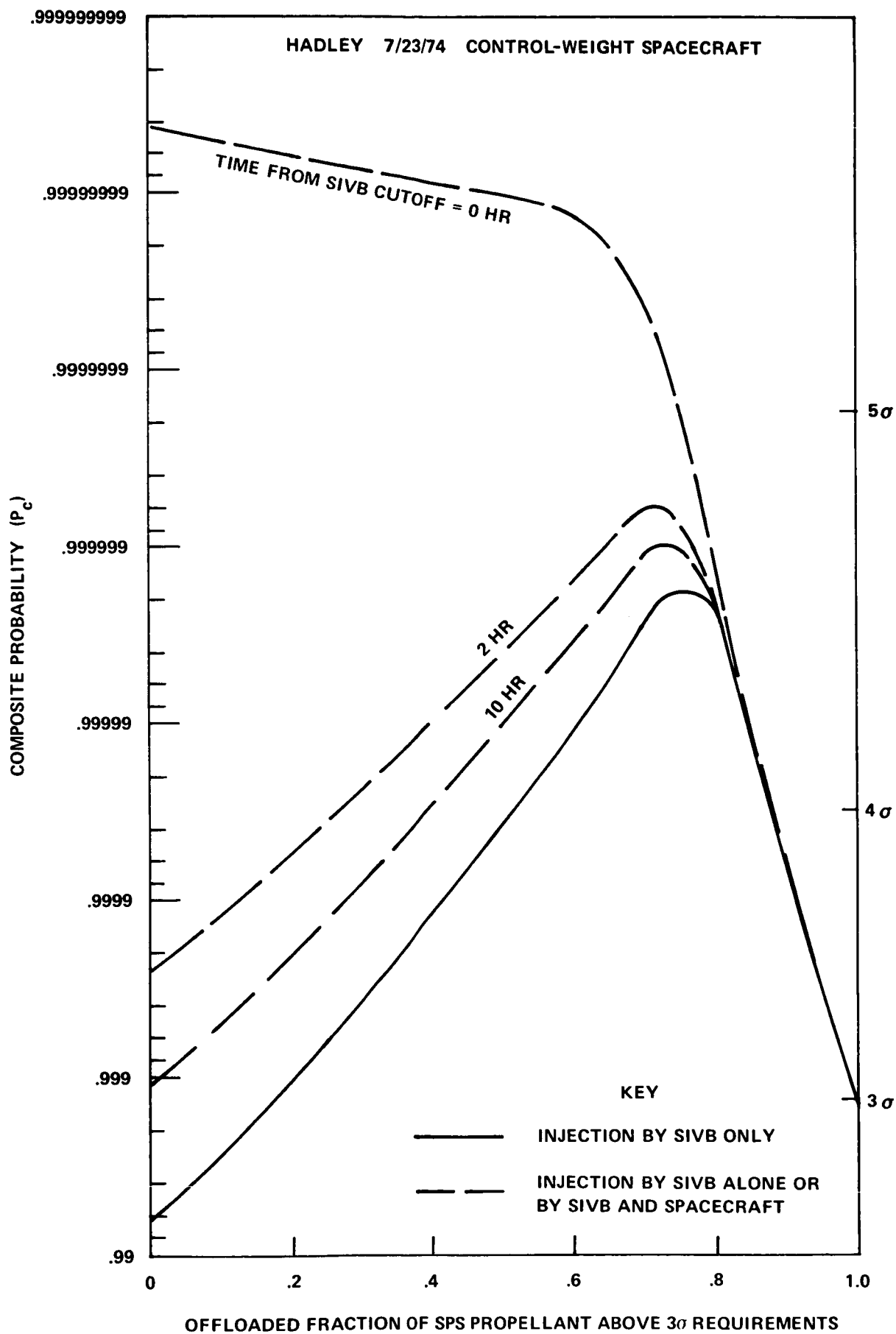


FIGURE 11 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

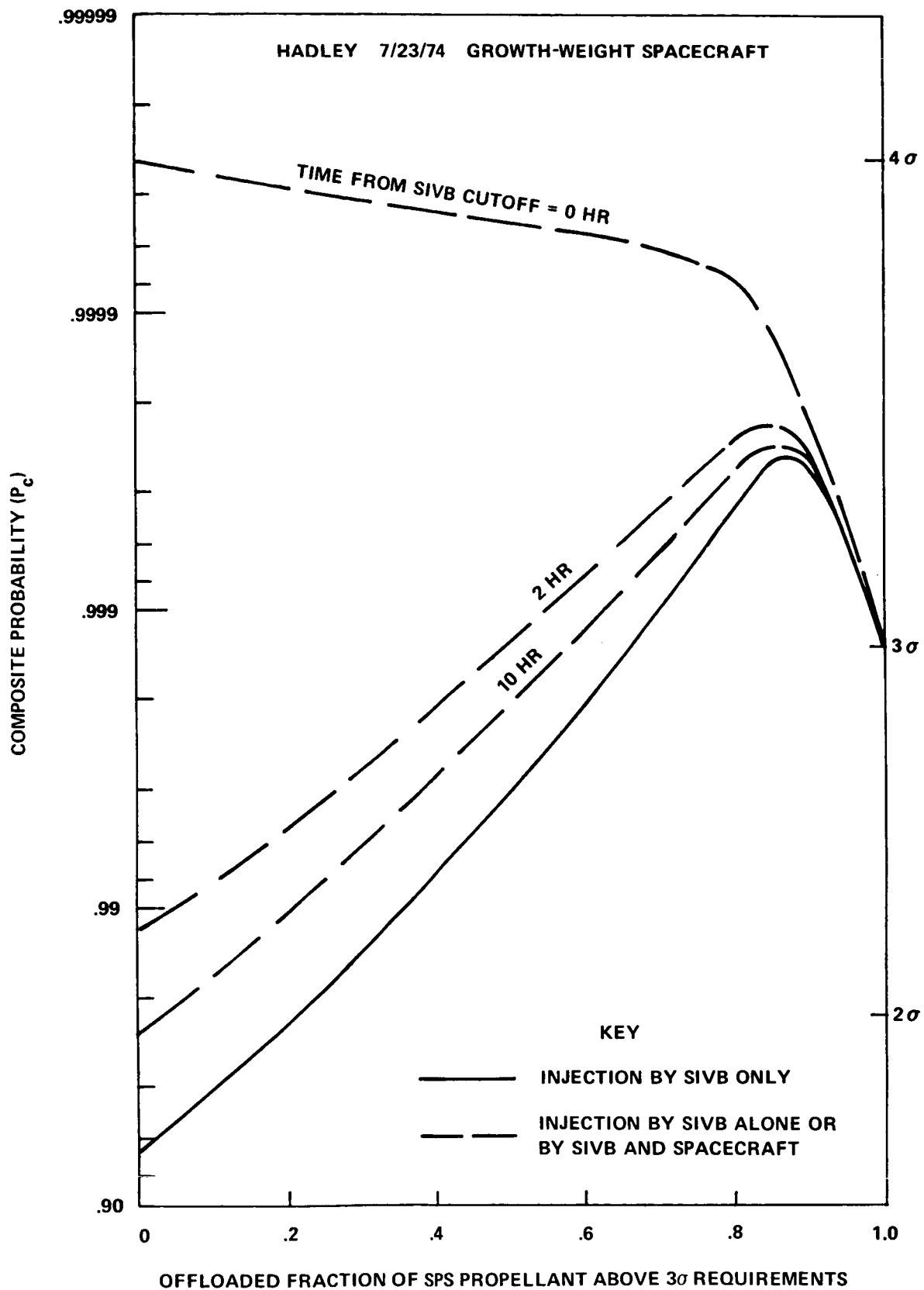


FIGURE 12 - COMPOSITE PROBABILITY vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

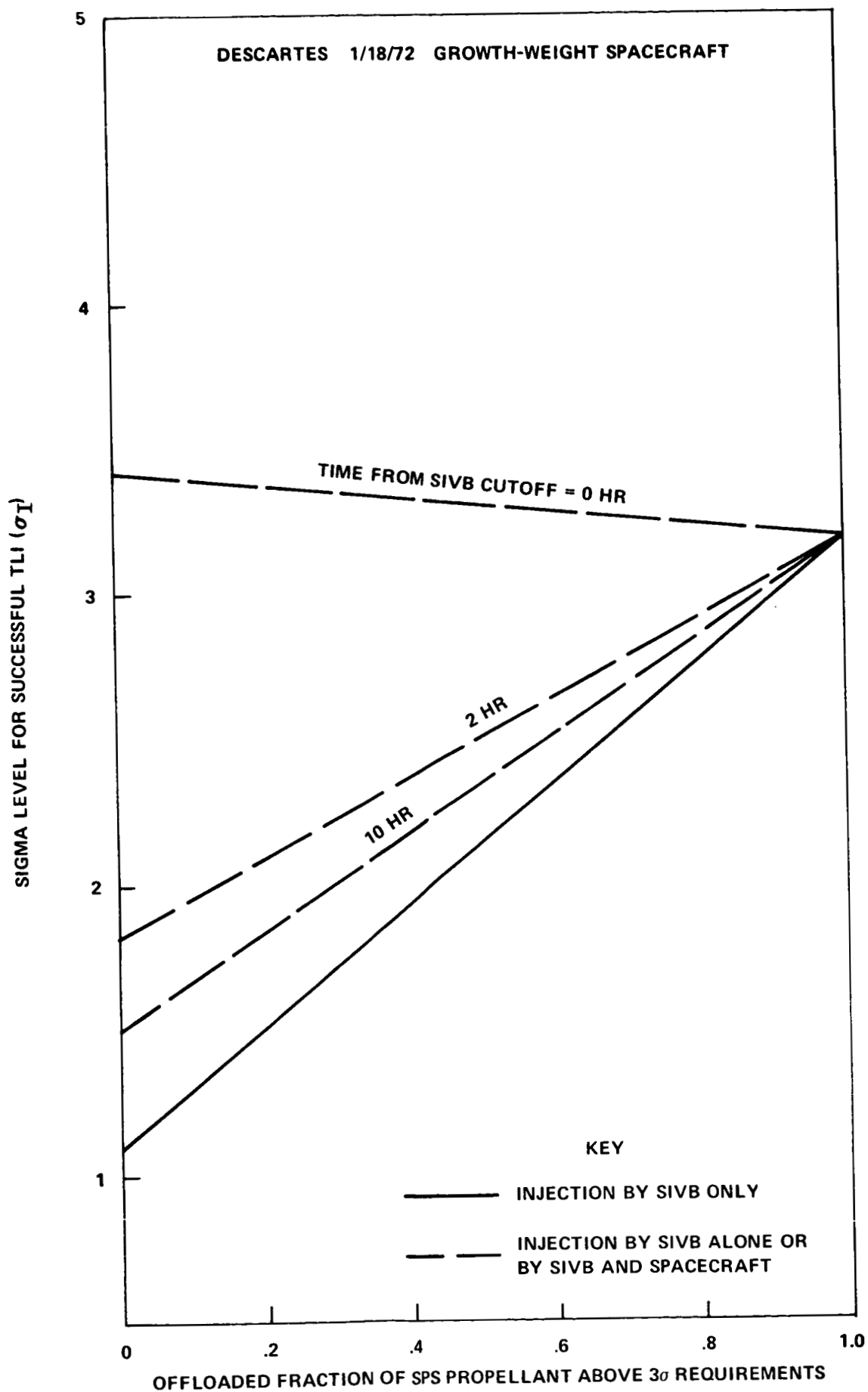


FIGURE 13 - INJECTION SIGMA PROBABILITY LEVEL vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

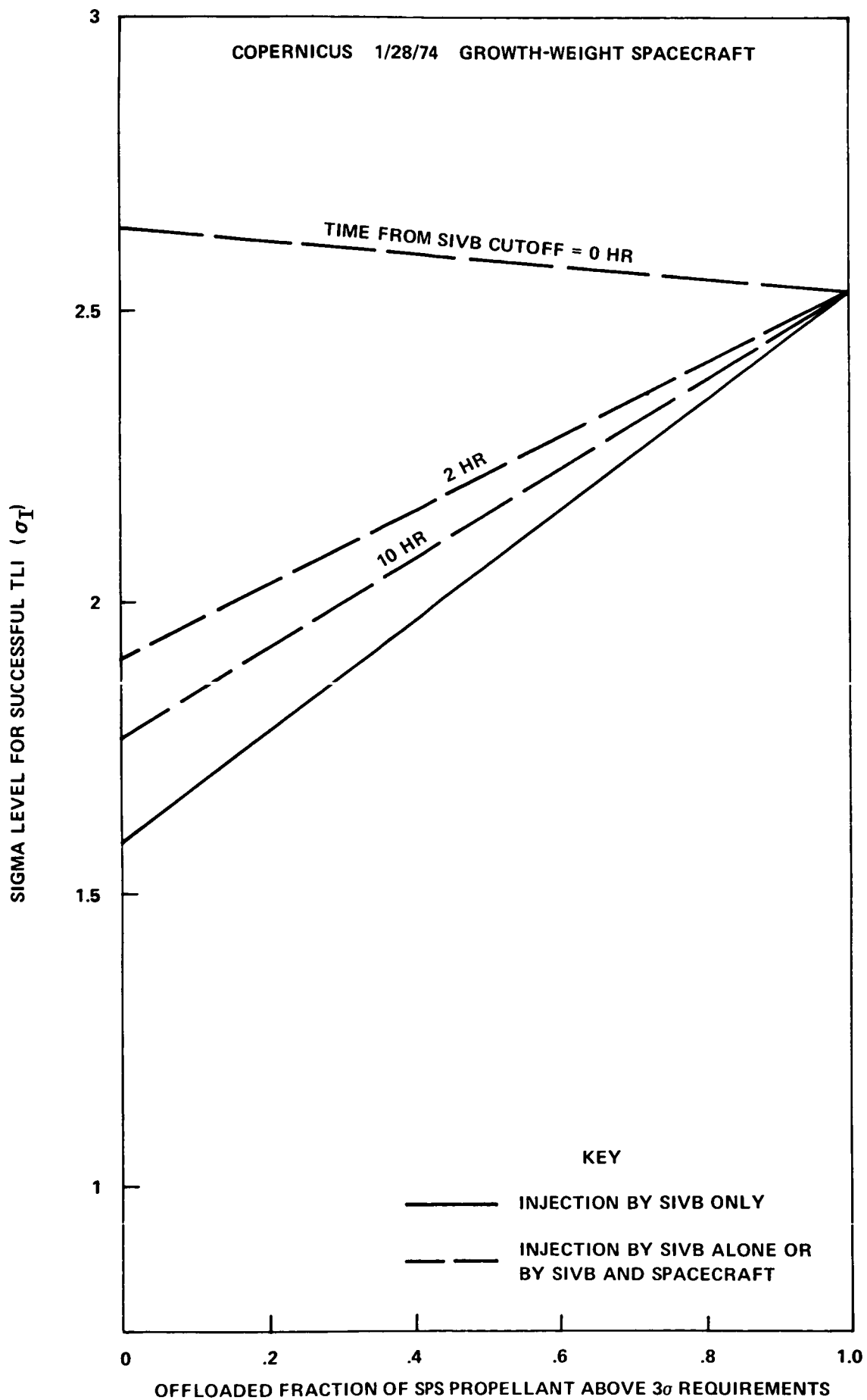


FIGURE 14 - INJECTION SIGMA PROBABILITY LEVEL vs. SPS PROPELLANT OFFLOADING
(INJECTION BY SIVB AND SPACECRAFT)

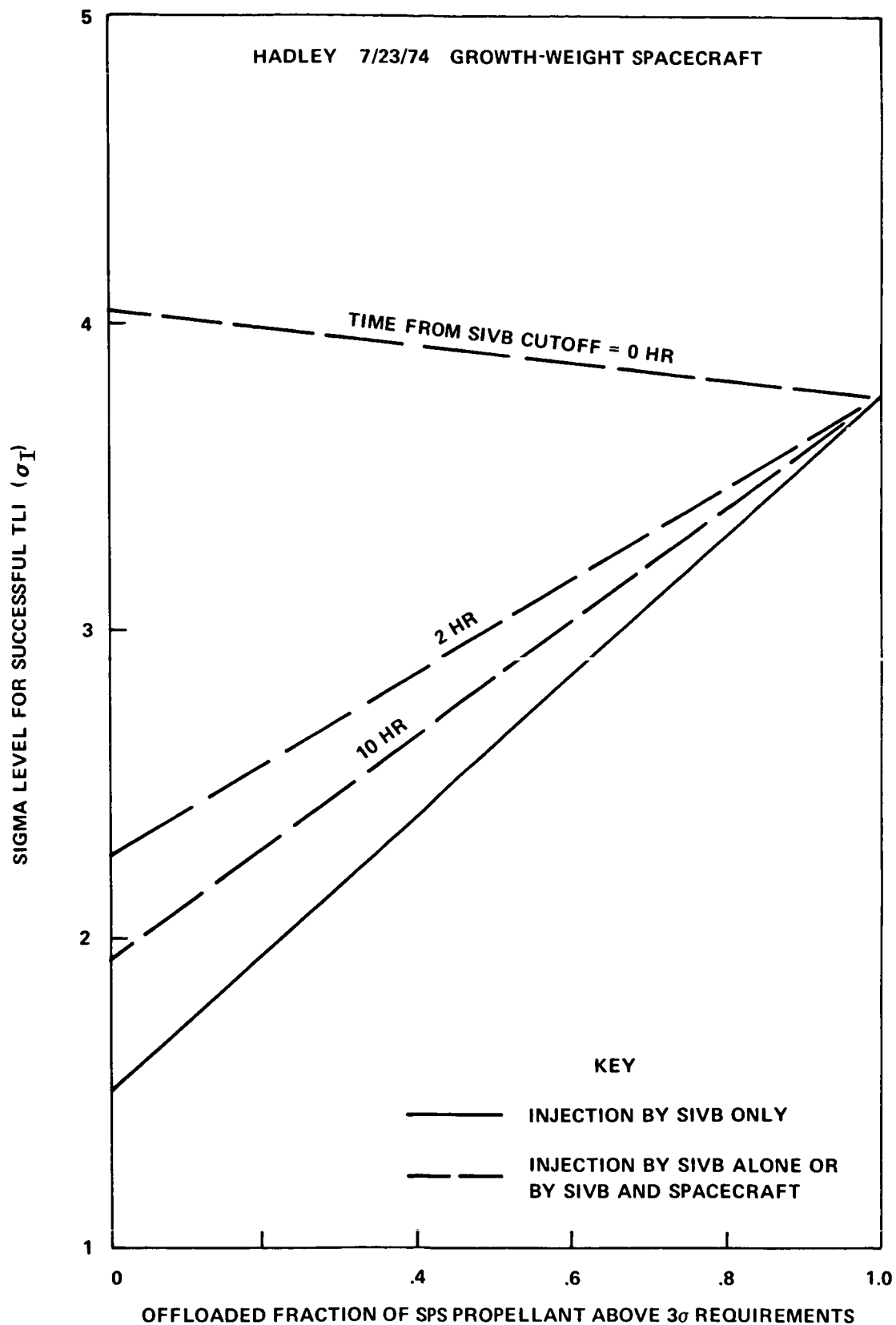


FIGURE 15 - INJECTION SIGMA PROBABILITY LEVEL vs. SPS PROPELLANT OFFLOADING (INJECTION BY SIVB AND SPACECRAFT)

DESCARTES 1/18/72 CONTROL-WEIGHT SPACECRAFT

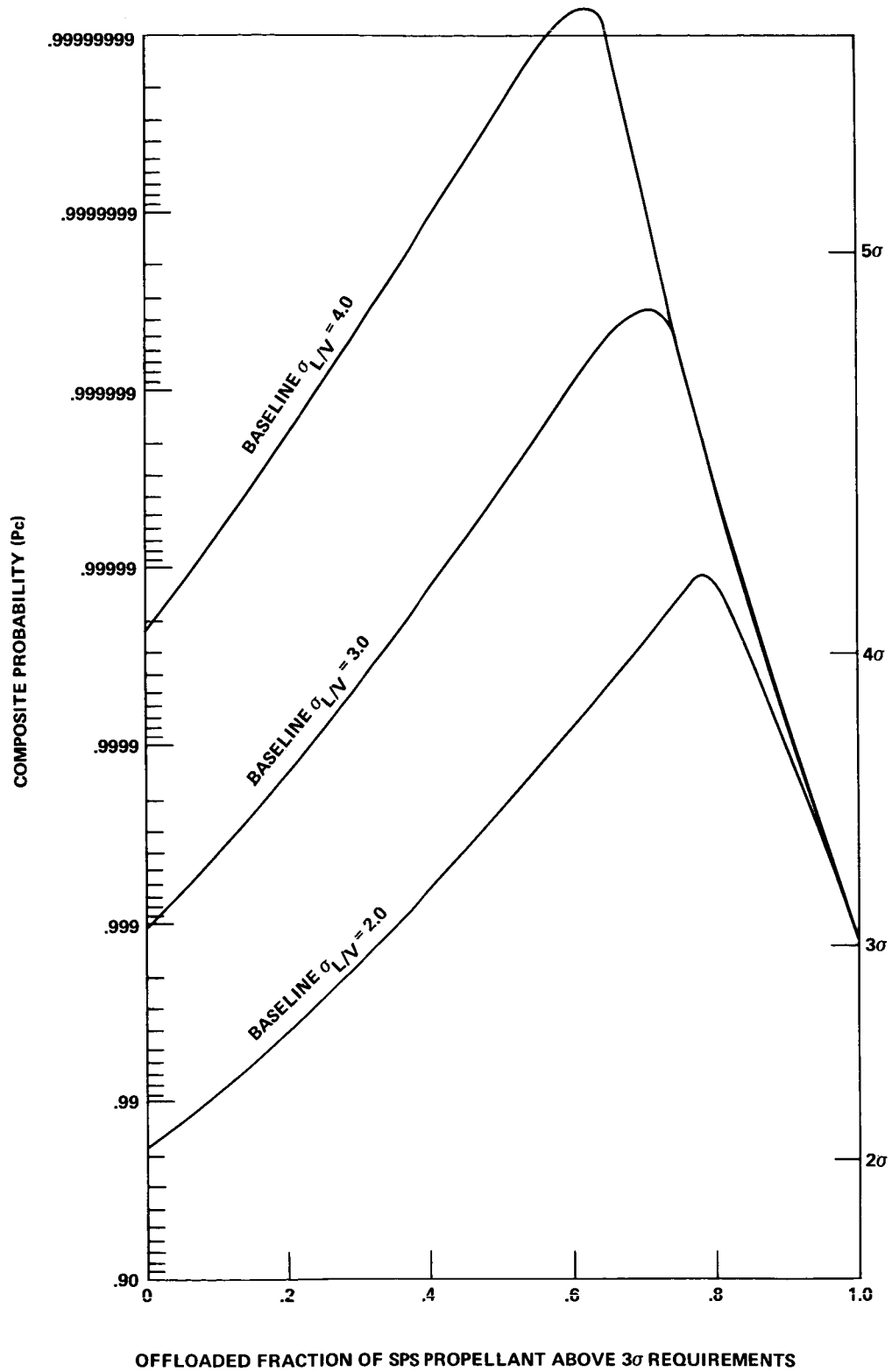


FIGURE 16 - COMPOSITE PROBABILITY VS. SPS PROPELLANT OFFLOADING FOR VARIATIONS IN BASELINE LAUNCH VEHICLE SIGMA LEVEL

DESCARTES 1/18/72 GROWTH-WEIGHT SPACECRAFT

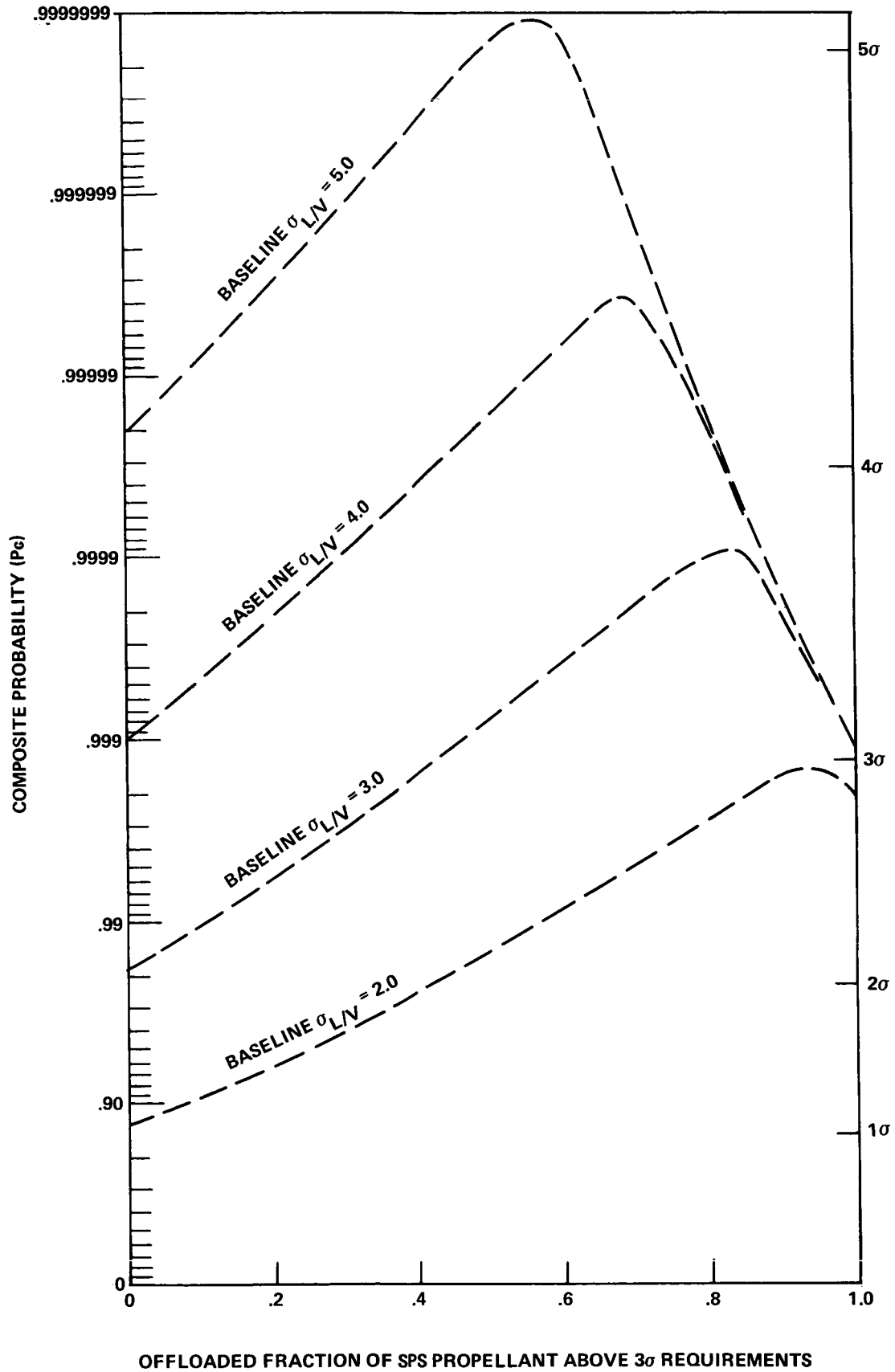


FIGURE 17 - COMPOSITE PROBABILITY VS. SPS PROPELLANT OFFLOADING FOR VARIATIONS IN BASELINE LAUNCH VEHICLE SIGMA LEVEL.

COPERNICUS 1/28/74 CONTROL-WEIGHT SPACECRAFT

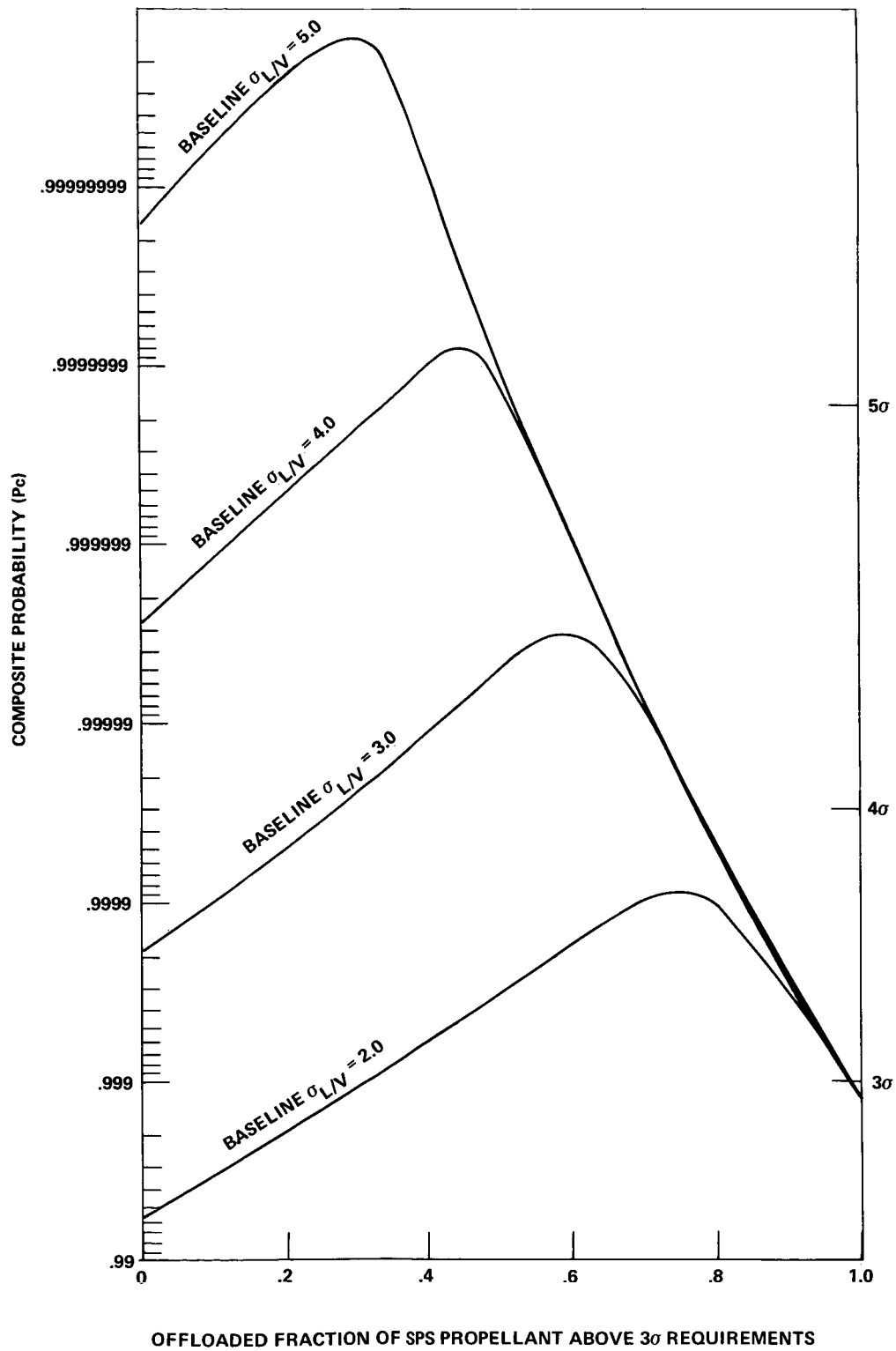


FIGURE 18 - COMPOSITE PROBABILITY VS. SPS PROPELLANT OFFLOADING FOR VARIATIONS IN BASELINE LAUNCH VEHICLE SIGMA LEVEL

COPERNICUS 1/28/74 GROWTH-WEIGHT

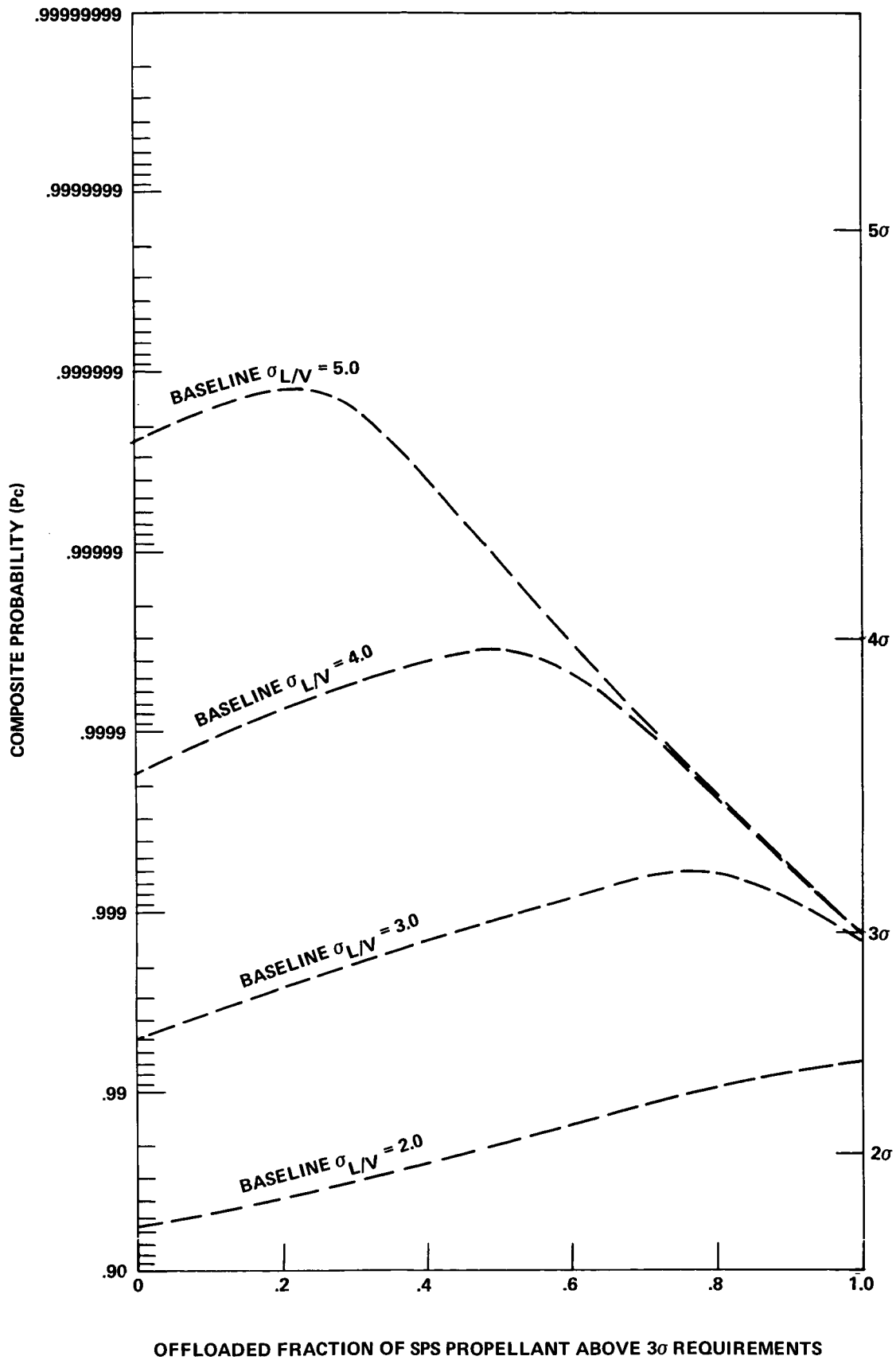


FIGURE 19 - COMPOSITE PROBABILITY VS. SPS PROPELLANT OFFLOADING FOR VARIATIONS IN BASELINE LAUNCH VEHICLE SIGMA LEVEL

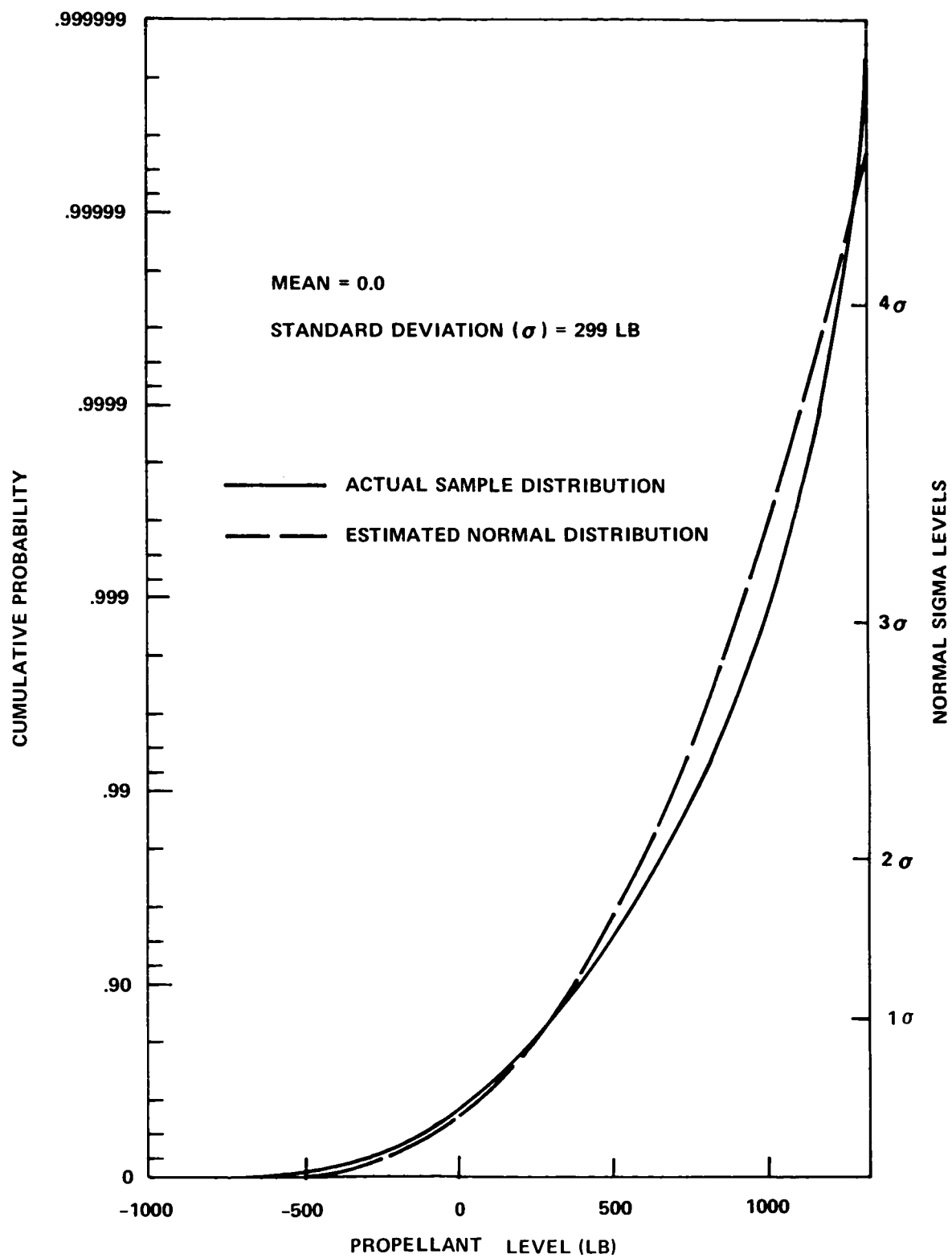


FIGURE 20 - CUMULATIVE PROBABILITY FUNCTION FOR SAMPLE SPACECRAFT DISPERSION BUDGET.